The Global Integrated Monetary and Fiscal Model (GIMF)

Technical Appendix

Michael Kumhof, International Monetary Fund
Douglas Laxton, International Moneary Fund

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1 MODEL OVERVIEW

The world consists of $\tilde{N}$ countries. The domestic economy is indexed by 1 and foreign economies by $j = 2, ..., \tilde{N}$. In our exposition we will ignore country indices except when interactions between multiple countries are concerned. It is understood that all parameters except population growth n and technology growth g can differ across countries. Figure 1 illustrates the flow of goods and factors for the two country case.

Countries are populated by two types of households, both of which consume final retailed output and supply labor to unions. First, there are overlapping generations households with finite planning horizons as in Blanchard (1985). Each of these agents faces a constant probability of death $(1-\theta(j))$ in each period, which implies an average planning horizon of $1/(1-\theta(j))$.[1] In each period, $N(j)n^t(1-\psi(j))\left(1-\frac{\theta(j)}{n}\right)$ of such individuals are born, where $N(j)$ indexes absolute population sizes in period 0 and $\psi(j)$ is the share of liquidity constrained agents. Second, there are liquidity constrained households who do not have access to financial markets, and who consequently are forced to consume their after tax income in every period. The number of such agents born in each period is $N(j)n^t\psi(j)\left(1-\frac{\theta(j)}{n}\right)$. Aggregation over different cohorts of agents implies that the total numbers of agents in country j is $N(j)n^t$. For computational reasons we do not normalize world population to one, especially when we analyze a small open economy. In that case we assume $N(1) = 1$, and set $N(j)$ such that $N(1)/\Sigma_{j=2}^{\tilde{N}}N(j)$ equals the share of country 1 agents in the world population. In addition to the probability of death households also experience labor productivity that declines at a constant rate over their lifetimes. This simplified treatment of lifecycle income profiles is justified by the absence of explicit demographics in our model, and adds another powerful channel through which fiscal policies can have non-Ricardian effects. Households of both types are subject to uniform labor income, consumption and lump-sum taxes. We will denote variables pertaining to these two groups of households by OLG and LIQ.

Firms are managed in accordance with the preferences of their owners, myopic OLG households, and they therefore also have finite planning horizons. Each country's primary production is carried

[1] In general we allow for the possibility that agents may be more myopic than what would be suggested by a planning horizon based on a biological probability of death.

out by manufacturers producing tradable and nontradable goods. Manufacturers buy capital services from entrepreneurs, labor from monopolistically competitive unions, and oil from the world oil market. They are subject to nominal rigidities in price setting as well as real rigidities in labor hiring and in the use of oil. Entrepreneurs finance their capital holdings using a combination of external and internal financing. They are subject to a capital income tax, and they buy physical capital from capital goods producers that are subject to investment adjustment costs. Unions are subject to nominal wage rigidities and buy labor from households. Manufacturers' domestic sales go to domestic distributors. Their foreign sales go to import agents that are domestically owned but located in each export destination country. Import agents in turn sell their output to foreign distributors. When the pricing-to-market assumption is made these import agents are subject to nominal rigidities in foreign currency. Distributors first assemble nontradable goods and domestic and foreign tradable goods, where changes in the volume of imported inputs are subject to an adjustment cost. This private sector output is then combined with a publicly provided capital stock (infrastructure) as an essential further input. This capital stock is maintained through government investment expenditure that is financed by tax revenue. The combined final domestic output is then sold to consumption goods producers, investment goods producers, and import agents located abroad. Consumption and investment goods producers in turn combine domestic and foreign output to produce final consumption and investment goods. Foreign output is purchased through a second set of import agents that can price to the domestic market, and again changes in the volume of imported goods are subject to an adjustment cost. This second layer of trade at the level of final output is critical for allowing the model to produce the high trade to GDP ratios typically observed in small, highly open economies. Consumption goods output is sold to retailers and the government, while investment goods output is sold domestic capital goods producers and the government. Consumption and investment goods producers are subject to another layer of nominal rigidities in price setting. This cascading of nominal rigidities from upstream to downstream sectors has important consequences for the behavior of aggregate inflation. Retailers, who are also monopolistically competitive, face real instead of nominal rigidities. While their output prices are flexible they find it costly to rapidly adjust their sales volume. This feature contributes to generating inertial consumption dynamics.[2]

[2] The alternative of using habit persistence to generate consumption inertia is not available

The world economy experiences a constant positive trend technology growth rate $g = T_t/T_{t-1}$, where T_t is the level of labor augmenting world technology, and a constant positive population growth rate n. When the model's real variables, say x_t, are rescaled, we divide by the level of technology T_t and by population, but for the latter we divide by n^t only, meaning real figures are not in per capita terms but rather in absolute terms adjusted for technology and population growth. We use the notation $\check{x}_t = x_t/(T_t n^t)$, with the steady state of $\check{x}_t$ denoted by $\bar{x}$. An exception to this is quantities of labor, which are only rescaled by n^t.

Asset markets are incomplete. There is complete home bias in government debt, which takes the form of nominally non-contingent one-period bonds denominated in domestic currency. The only assets traded internationally are nominally non-contingent one-period bonds denominated in the currency of $\tilde{N}$. There is also complete home bias in ownership of domestic firms. In addition equity is not traded in domestic financial markets, instead households receive lump-sum dividend payments. This assumption is required to support our assumption that firm and not just household preferences feature myopia.

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in our setup. This is because we face two constraints in our choice of household preferences. The first is that preferences must be consistent with balanced growth. The second is the necessity of being able to aggregate across generations of households. We are left with preferences that, while commonly used, do not allow for a powerful role of habit persistence.

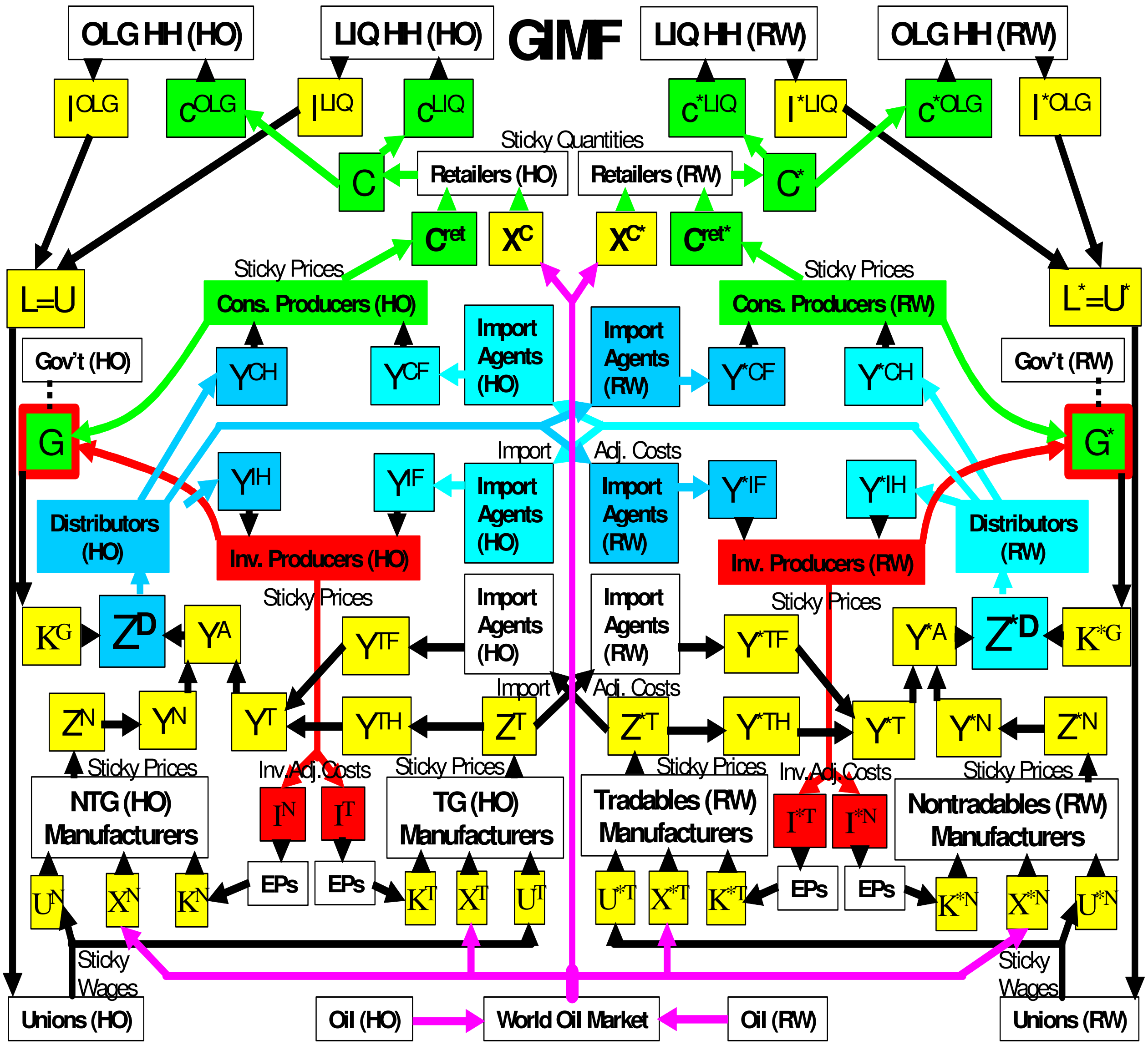
GIMF
OLG HH (HO)
LIQ HH (HO)
LIQ HH (RW)
OLG HH (RW)
Sticky Quantities
Retailers (HO)
Retailers (RW)
Sticky Prices
Cons. Producers (HO)
Cons. Producers (RW)
Import Agents (HO)
Import Agents (RW)
Gov't (HO)
Gov't (RW)
Import Adj. Costs
Distributors (HO)
Distributors (RW)
Inv. Producers (HO)
Inv. Producers (RW)
Inv. Adj. Costs
NTG (HO) Manufacturers
TG (HO) Manufacturers
Tradables (RW) Manufacturers
Nontradables (RW) Manufacturers
EPs
Sticky Wages
Unions (HO)
Oil (HO)
World Oil Market
Oil (RW)
Unions (RW)

Figure 1

2 Overlapping Generations Households

We first describe the optimization problem of OLG households. A representative member of this group and of age a derives utility at time t from consumption $c_{a,t}^{OLG}$, leisure $(S_t^L - \ell_{a,t}^{OLG})$ (where S_t^L is the stochastic time endowment, with a mean of one), and real balances $(M_{a,t}/P_t^R)$ (where P_t^R is the retail price index). The lifetime expected utility of a representative household of age a at time t has the form

$$E_t \sum_{s=0}^{\infty} (\beta_t \theta)^s \left[\frac{1}{1-\gamma} \left(\left(c_{a+s,t+s}^{OLG}\right)^{\eta^{OLG}} \left(S_t^L - \ell_{a+s,t+s}^{OLG}\right)^{1-\eta^{OLG}} \right)^{1-\gamma} + \frac{u^m}{1-\gamma} \left(\frac{M_{a+s,t+s}}{P_{t+s}^R} \right)^{1-\gamma} \right] , \tag{1}$$

where E_t is the expectations operator, $\theta < 1$ is the degree of myopia, $\gamma > 0$ is the coefficient of relative risk aversion, $0 < \eta^{OLG} < 1$[3], $u^m > 0$, and β_t is the (stochastic) discount factor. As for money demand, in the following analysis we will only consider the case of the cashless limit advocated by Woodford (2003), where $u^m \longrightarrow 0$. As a result the optimality conditions for money will be ignored throughout our analysis. Note that this does not involve a great loss of generality in our case, and in fact it has one major advantage. The reason is that the combination of separable money in the utility function and monetary policy specified as an interest rate rule implies that the money demand equation becomes redundant and that inflation is not directly distortionary for the consumption-leisure decision. But money also has a fiscal role through the government budget constraint, and any reduction in inflation tax revenue must be accompanied by an offsetting increase in other forms of distortionary taxation.[4] Because of this indirect distortionary effect, an increase in inflation in this model would actually reduce overall distortions unless we consider the case of the cashless limit, in which case inflation causes no distortions in either direction.

[3] For flexible model calibration we allow for the possibility that OLG households attach a different weight η^{OLG} to consumption than liquidity constrained households. This allows us to model both groups as working during an equal share of their time endowment in steady state, while OLG households have much higher consumption due to their accumulated wealth.

[4] Except for the special case of lump-sum taxation.

Consumption $c_{a,t}^{OLG}$ is given by a CES aggregate over retailed consumption goods varieties $c_{a,t}^{OLG}(i)$, with elasticity of substitution σ_R:

$$c_{a,t}^{OLG} = \left(\int_0^1 \left(c_{a,t}^{OLG}(i) \right)^{\frac{\sigma_R - 1}{\sigma_R}} di \right)^{\frac{\sigma_R}{\sigma_R - 1}} . \tag{2}$$

This gives rise to a demand for individual varieties

$$c_{a,t}^{OLG}(i) = \left(\frac{P_t^R(i)}{P_t^R} \right)^{-\sigma_R} c_{a,t}^{OLG} , \tag{3}$$

where $P_t^R(i)$ is the retail price of variety i, and the aggregate retail price level P_t^R is given by

$$P_t^R = \left(\int_0^1 \left(P_t^R(i) \right)^{1-\sigma_R} di \right)^{\frac{1}{1-\sigma_R}} . \tag{4}$$

A household can hold two types of bonds. The first bond type is domestic bonds denominated in domestic currency. Such bonds are issued either by the domestic government $B_{a,t}$ or, in the case of GIMF with a Financial Accelerator, by banks lending to the nontradables or tradables sector, $B_{a,t}^N + B_{a,t}^T$. The second bond type is foreign bonds denominated in the currency of country $\tilde{N}$, $F_{a,t}$. The nominal exchange rate vis-a-vis $\tilde{N}$ is denoted by $\mathcal{E}_t$, and $\mathcal{E}_t F_{a,t}$ are nominal net foreign asset holdings in terms of domestic currency. In each case the time subscript t denotes financial claims held from period t to period $t+1$. Gross nominal interest rates on domestic and foreign currency denominated assets held from t to $t+1$ are $i_t/(1+\xi_t^b)$ and $i_t(\tilde{N})(1+\xi_t^f)$. For domestic bonds, i_t is the nominal interest rate paid by the domestic government and ξ_t^b is a domestic risk premium, with $\xi_t^b < 0$ characterizing a situation where the private sector faces a larger marginal funding rate than the public sector. For foreign bonds, $i_t(\tilde{N})$ is the nominal interest rate determined in $\tilde{N}$, and ξ_t^f is a foreign exchange risk premium. Both risk premia are external to the household's asset accumulation decision, and are payable to a financial intermediary that redistributes the proceeds in a lump-sum fashion either to foreigners or to domestic households. The functional form of the foreign exchange risk premium is given by

$$\xi_t^f = y_1 + \frac{y_2}{\left(cagdp_t^{filt} - y_4 \right)^{y_3}} + S_t^{fx} , \tag{5}$$

$$cagdp_t^{filt} = E_t \left(\Sigma_{k=k_l^{ca}}^{k_h^{ca}} 100 \frac{ca_{t+j}}{gdp_{t+j}} \right) / \left(k_h^{ca} - k_l^{ca} + 1 \right) , \tag{6}$$

where S_t^{fx} is a mean zero risk premium shock, $y_1 - y_4$ are parameters, y_1 is constrained to generate a zero premium at a zero current account by the condition $y_1 = -y_2/(-y_4)^{y_3}$, and $cagdp_t^{filt}$ is a moving average of past and future current account to GDP ratios, with k_h^{ca} the maximum lead and k_l^{ca} the maximum lag. We have found this functional form to be more suitable for applied work than conventional quadratic specifications because it is asymmetric, allowing for a steeply increasing risk premium at large current account deficits.

The functional form of the domestic risk premium can similarly be made to depend on the government debt to GDP ratio when it is intended to highlight the effect of government borrowing levels on domestic interest rates. But it can also be treated as an exogenous stochastic process when the emphasis is on shocks to the interest rate margin between the policy rate and the rate at which the private sector can access the domestic capital market. For example, recent financial markets events may be partly characterized by a persistent negative shock to ξ_t^b.

Participation by households in financial markets requires that they enter into an insurance contract with companies that pay a premium of $\frac{(1-\theta)}{\theta}$ on a household's financial wealth for each period in which that household is alive, and that encash the household's entire financial wealth in the event of his death.[5]

Apart from returns on financial assets, households also receive labor and dividend income. Households sell their labor to "unions" that are competitive in their input market and monopolistically competitive in their output market, vis-à-vis manufacturing firms. The productivity of a household's labor declines throughout his lifetime, with productivity $\Phi_{a,t} = \Phi_a$ of age group a given by

$$\Phi_a = \kappa \chi^a \ , \tag{7}$$

where $\chi < 1$. The overall population's average productivity is assumed without loss of generality to be equal to one. Household pre-tax nominal labor income is therefore $W_t \Phi_{a,t} \ell_{a,t}^{OLG}$. Dividends are received in a lump-sum fashion from all firms in the nontradables (N) and tradables (T) manufacturing sectors, from the distribution (D), consumption goods distribution (C) and investment goods distribution (I) sectors, from the retail (R) sector and the import agent (M) sector, from all unions (U) in the labor market, from domestic (X) and foreign (F) raw materials producers,

[5] The turnover in the population is assumed to be large enough that the income receipts of the insurance companies exactly equal their payouts.

from capital goods producers (K), and from entrepreneurs (EP), with after-tax nominal dividends received from firm/union i denoted by $D^j_{a,t}(i)$, $j = N, T, D, C, I, R, U, M, X, F, K, EP$. OLG households are liable to pay lump-sum transfers $\tau^{OLG}_{T_{a,t}}$ to the government, which in turn redistributes them to the relatively less well off LIQ agents. Household labor income is taxed at the rate $\tau_{L,t}$, and consumption is taxed at the rate $\tau_{c,t}$. In addition there are lump-sum taxes $\tau^{ls}_{a,t}$ and transfers $\Upsilon_{a,t}$ paid to/from the government.[6] It is assumed that retailers face costs of rapidly adjusting their sales volume. To limit these costs they therefore offer incentives (or disincentives) that are incorporated into the effective retail purchase price P^R_t. The consumption tax $\tau_{c,t}$ is however assumed to be payable on the pre-incentive price P^C_t.[7] P^C_t is the marginal cost of retailers, who combine the output of consumption goods producers, with price level P_t, with raw materials used directly by consumers, with price level P^X_t. We choose P_t as our numeraire, and denote the real wage by $w_t = W_t/P_t$, the relative price of any good x by $p^x_t = P^x_t/P_t$, gross inflation for any good x by $\pi^x_t = P^x_t/P^x_{t-1}$, and gross nominal exchange rate depreciation by $\varepsilon_t = \mathcal{E}_t/\mathcal{E}_{t-1}$.[8]

The household's budget constraint in nominal terms is

$$P^R_t c^{OLG}_{a,t} + P^C_t c^{OLG}_{a,t} \tau_{c,t} + P_t \tau^{ls}_{a,t} + B_{a,t} + B^N_{a,t} + B^T_{a,t} + \mathcal{E}_t F_{a,t} \tag{8}$$

$$= \frac{1}{\theta} \left[\frac{i_{t-1}}{(1+\xi^b_{t-1})} \left(B_{a-1,t-1} + B^N_{a-1,t-1} + B^T_{a-1,t-1} \right) + i_{t-1}(\tilde{N}) \mathcal{E}_t F_{a-1,t-1} (1+\xi^f_{t-1}) \right]$$

$$+ W_t \Phi_{a,t} \ell^{OLG}_{a,t} (1-\tau_{L,t}) + \sum_{j=N,T,D,C,I,R,U,M,X,F,K,EP} \int_0^1 D^j_{a,t}(i) di - \tau^{OLG}_{T_{a,t}} + P_t \Upsilon_{a,t} \; .$$

The OLG household maximizes (1) subject to (2), (7) and (8). The derivation of the first-order conditions for each generation, and aggregation across generations, is discussed in detail in Appendices A and B. Aggregation takes account of the size of each age cohort at the time of birth, and of the remaining size of each generation. Using the example of overlapping generations households'

[6] It is sometimes convenient to keep these two items separate when trying to account for a country's overall fiscal structure, and when allowing for targeted transfers to LIQ agents.

[7] Without this assumption consumption tax revenue could become too volatile in the short run.

[8] We adopt the convention throughout the paper that all nominal price level variables are written in upper case letters, and that all relative price variables are written in lower case letters.

consumption, we have

$$c_t^{OLG} = Nn^t(1-\psi)\left(1-\frac{\theta}{n}\right)\Sigma_{a=0}^{\infty}\left(\frac{\theta}{n}\right)^a c_{a,t}^{OLG} \ . \tag{9}$$

This also has implications for the intercept parameter κ of the age-specific productivity distribution. Under the assumption of an average productivity of one, and for given parameters χ and θ, we obtain $\kappa = (n-\theta\chi)/(n-\theta)$. Several of the optimality conditions that need to be aggregated are, or are derived from, nonlinear Euler equations. In such conditions, aggregation requires nonlinear transformations that are only valid under certainty equivalence. Tractable aggregate consumption optimality conditions therefore only exist for the cases of perfect foresight and of first-order approximations. For our purposes this is not problematic as all applications of GIMF will use at most log-linear approximations. However, for the purpose of exposition we find it preferable to present optimality conditions in nonlinear form. We therefore adopt the notation $\tilde{E}_t$ to denote an expectations operator that is understood in this fashion.

The first-order conditions for the goods varieties and for the consumption/leisure choice are given by

$$\check{c}_t^{OLG}(i) = \left(\frac{P_t^R(i)}{P_t^R}\right)^{-\sigma_R}\check{c}_t^{OLG} \ , \tag{10}$$

$$\frac{\check{c}_t^{OLG}}{N(1-\psi)S_t^L - \check{\ell}_t^{OLG}} = \frac{\eta^{OLG}}{1-\eta^{OLG}}\check{w}_t\frac{(1-\tau_{L,t})}{(p_t^R+p_t^C\tau_{c,t})} \ . \tag{11}$$

The arbitrage condition for foreign currency bonds (the uncovered interest parity relation) is given by

$$i_t = i_t(\tilde{N})(1+\xi_t^f)(1+\xi_t^b)\tilde{E}_t\varepsilon_{t+1} \ . \tag{12}$$

The consumption Euler equation on the other hand cannot be directly aggregated across generations. For each generation we have

$$E_t c_{a+1,t+1} = E_t j_t c_{a,t} \ , \tag{13}$$

$$j_t = \left(\frac{\beta}{\check{r}_{t+1}}\right)^{\frac{1}{\gamma}}\left(\frac{p_t^R+p_t^C\tau_{c,t}}{p_{t+1}^R+p_{t+1}^C\tau_{c,t+1}}\right)^{\frac{1}{\gamma}}\left(\chi g\frac{\check{w}_{t+1}(1-\tau_{L,t+1})(p_t^R+p_t^C\tau_{c,t})}{\check{w}_t(1-\tau_{L,t})(p_{t+1}^R+p_{t+1}^C\tau_{c,t+1})}\right)^{(1-\eta^{OLG})(1-\frac{1}{\gamma})} . \tag{14}$$

Here we have used the notation

$$\check{r}_t = \frac{i_{t-1}}{\pi_t \left(1 + \xi^b_{t-1}\right)} = \frac{r_t}{\left(1 + \xi^b_{t-1}\right)} \ . \tag{15}$$

We introduce some additional notation. The production based real exchange rate vis-a-vis $\tilde{N}$ is $e_t = (\mathcal{E}_t P_t(\tilde{N}))/P_t$, where $P_t(\tilde{N})$ is the price of final output in $\tilde{N}$. We adopt the convention that each nominal asset is deflated by the final output price index of the currency of its denomination, so that real domestic bonds are $b_t = B_t/P_t$ and real foreign bonds are $f_t = F_t/P_t(\tilde{N})$. The real interest rate in terms of final output payable by the government is $r_t = i_t/\pi_{t+1}$, while the real interest rate payable by the private sector is $\check{r}_t = (i_t/\pi_{t+1}) / \left(1 + \xi^b_t\right)$. The subjective and market nominal discount factors are given by

$$\tilde{R}_{t,s} = \Pi^s_{l=1} \frac{\theta \left(1 + \xi^b_{t+l-1}\right)}{i_{t+l-1}} \text{ for } s > 0 \ (= 1 \text{ for } s = 0) \ , \tag{16}$$

$$R_{t,s} = \Pi^s_{l=1} \frac{\left(1 + \xi^b_{t+l-1}\right)}{i_{t+l-1}} \text{ for } s > 0 \ (= 1 \text{ for } s = 0) \ , \tag{17}$$

and the subjective and market real discount factors by

$$\tilde{r}_{t,s} = \Pi^s_{l=1} \frac{\theta}{\check{r}_{t+l-1}} \text{ for } s > 0 \ (= 1 \text{ for } s = 0) \ , \tag{18}$$

$$r_{t,s} = \Pi^s_{l=1} \frac{1}{\check{r}_{t+l-1}} \text{ for } s > 0 \ (= 1 \text{ for } s = 0) \ . \tag{19}$$

In each case the subjective discount factor incorporates an agent's probability of economic death, which ceteris paribus makes him value near term receipts more highly than receipts in the distant future.

We now discuss a key condition of GIMF, the optimal aggregate consumption rule of OLG households. The derivation of this condition is algebraically complex and is therefore presented in Appendix C. The final result expresses current aggregate consumption of OLG households as a function of their real aggregate financial wealth fw_t and human wealth hw_t, with the marginal propensity to consume of out of wealth given by $1/\Theta_t$. Human wealth is in turn composed of hw^L_t, the expected present discounted value of households' time endowments evaluated at the after-tax real wage, and hw^K_t, the expected present discounted value of capital or dividend income net of lump-sum transfer payments to the government. After rescaling by technology we have

$$\check{c}_t^{OLG}\Theta_t = \check{f}w_t + \check{h}w_t \ , \tag{20}$$

where

$$\check{f}w_t = \frac{1}{\pi_t g n}\left[\frac{i_{t-1}}{\left(1+\xi_{t-1}^b\right)}\left(\check{b}_{t-1} + \check{b}_{t-1}^N + \check{b}_{t-1}^T\right) + i_{t-1}(\tilde{N})(1+\xi_{t-1}^f)\varepsilon_t \check{f}_{t-1}e_{t-1}\right] \ , \tag{21}$$

$$\check{h}w_t = \check{h}w_t^L + \check{h}w_t^K \ , \tag{22}$$

$$\check{h}w_t^L = \left(N(1-\psi)(\check{w}_t(1-\tau_{L,t})S_t^L)\right) + \tilde{E}_t \frac{\theta\chi g}{\check{r}_{t+1}}\check{h}w_{t+1}^L \ , \tag{23}$$

$$\check{h}w_t^K = \left(\Sigma_{j=N,T,D,C,I,R,U,M,X,F,K,EP}\check{d}_t^j - \check{\tau}_{T,t}^{OLG} + \check{\Upsilon}_t^{OLG} - \check{\tau}_t^{ls,OLG}\right) + \tilde{E}_t \frac{\theta g}{\check{r}_{t+1}}\check{h}w_{t+1}^K \ , \tag{24}$$

$$\Theta_t = \frac{p_t^R + p_t^C \tau_{c,t}}{\eta^{OLG}} + \tilde{E}_t \frac{\theta j_t}{\check{r}_{t+1}}\Theta_{t+1} \ . \tag{25}$$

The intuition of (20) is key to GIMF. Financial wealth (21) is equal to the domestic government's and foreign households' *current* financial liabilities. For the government debt portion, the government services these liabilities through different forms of taxation, and these *future* taxes are reflected in the different components of human wealth (22) as well as in the marginal propensity to consume (25). But unlike the government, which is infinitely lived, an individual household factors in that he might not be around by the time higher future tax payments fall due. Hence *a household discounts future tax liabilities by a rate of at least* $\check{r}_t/\theta$*, which is higher than the market rate* $\check{r}_t$, as reflected in the discount factors in (23), (24) and (25). The discount rate for the labor income component of human wealth is even higher at $\check{r}_t/\theta\chi$, due to the decline of labor incomes over individuals' lifetimes.

A fiscal consolidation through higher taxes represents a tilting of the tax payment profile from the more distant future to the near future, so as to effect a reduction in the debt stock. The government has to respect its intertemporal budget constraint in effecting this tilting, and this means that the expected present discounted value of its future primary surpluses has to remain equal to the current debt $i_{t-1}b_{t-1}/\pi_t$ *when future surpluses are discounted at the market interest rate* r_t. But when individual households discount future taxes at a higher rate than the government, the same tilting of the tax profile represents a decrease in human wealth because it increases the expected value of future taxes for which the household expects to be responsible. This is true both for the direct effect of labor income taxes on labor income receipts, and for the indirect effect of corporate taxes on dividend receipts. For a given marginal propensity to consume, these reductions in human wealth lead to a reduction in consumption. Note that with $\xi_t^b < 0$ this effect is not only due to myopia but also to the borrowing spread between the public and private sectors.

The marginal propensity to consume $1/\Theta_t$ is, in the simplest case of logarithmic utility and exogenous labor supply, equal to $(1-\beta\theta)$. For the case of endogenous labor supply, household wealth can be used to either enjoy leisure or to generate purchasing power to buy goods. The main determinant of the split between consumption and leisure is the consumption share parameter η^{OLG}, which explains its presence in the marginal propensity to consume (25). While other forms of taxation affect the different components of wealth, the time profile of consumption taxes affects the marginal propensity to consume, reducing it with a balanced-budget shift of such taxes from the future to the present. The intertemporal elasticity of substitution $1/\gamma$ is another key parameter for the marginal propensity to consume. For the conventional assumption of $\gamma > 1$ the income effect of an increase in the real interest rate r is stronger than the substitution effect and tends to increase the marginal propensity to consume, thereby partly offsetting the contractionary effects of a higher r on human wealth $\check{h}w_t$. Larger γ therefore tends to give rise to larger interest rate changes in response to fiscal shocks.

3 Liquidity Constrained Households

The objective function of liquidity constrained (LIQ) households is assumed to be nearly identical to that of OLG households:[9]

$$E_t \sum_{s=0}^{\infty} (\beta\theta)^s \left[\frac{1}{1-\gamma} \left(\left(c^{LIQ}_{a+s,t+s} \right)^{\eta^{LIQ}} \left(S_t^L - \ell^{LIQ}_{a+s,t+s} \right)^{1-\eta^{LIQ}} \right)^{1-\gamma} \right] , \tag{26}$$

$$c^{LIQ}_{a,t} = \left(\int_0^1 \left(c^{LIQ}_{a,t}(i) \right)^{\frac{\sigma_R - 1}{\sigma_R}} di \right)^{\frac{\sigma_R}{\sigma_R - 1}} . \tag{27}$$

[9] The distinction of generations could be dropped as all agents must act identically.

These agents can consume at most their current income, which consists of their after tax wage income plus government transfers $\tau_{T_{a,t}}^{LIQ}$. Their budget constraint is

$$P_t^R c_{a,t}^{LIQ} + P_t^C c_{a,t}^{LIQ} \tau_{c,t} \leqq W_t \Phi_{a,t} \ell_{a,t}^{LIQ} (1 - \tau_{L,t}) + \tau_{T_{a,t}}^{LIQ} + \Upsilon_{a,t}^{LIQ} - \tau_{a,t}^{ls,LIQ} \; . \tag{28}$$

The aggregated first-order conditions for this problem, after rescaling by technology, are

$$\check{c}_t^{LIQ}(i) = \left(\frac{P_t^R(i)}{P_t^R} \right)^{-\sigma_R} \check{c}_t^{LIQ} \; , \tag{29}$$

$$\check{c}_t^{LIQ} (p_t^R + p_t^C \tau_{c,t}) = \check{w}_t \ell_t^{LIQ} (1 - \tau_{L,t}) + \check{\tau}_{T,t}^{LIQ} + \check{\Upsilon}_t^{LIQ} - \check{\tau}_t^{ls,LIQ} \; , \tag{30}$$

$$\frac{\check{c}_t^{LIQ}}{N\psi S_t^L - \check{\ell}_t^{LIQ}} = \frac{\eta^{LIQ}}{1 - \eta^{LIQ}} \check{w}_t \frac{(1 - \tau_{L,t})}{(p_t^R + p_t^C \tau_{c,t})} \; . \tag{31}$$

GIMF also allows for an alternative version where equation (31) is dropped and is replaced with an exogenous labor supply, the so-called “rule of thumb consumer”.

4 Aggregate Household Sector

To obtain aggregate consumption demand and labor supply we simply add the respective optimality quantities of the different consumers in the economy. For GIMF without a Financial Accelerator these are OLG and LIQ households:

$$\check{C}_t = \check{c}_t^{OLG} + \check{c}_t^{LIQ} \; , \tag{32}$$

$$\check{L}_t = \check{\ell}_t^{OLG} + \check{\ell}_t^{LIQ} \; . \tag{33}$$

5 Manufacturers

There is a continuum of manufacturing firms indexed by $i \in [0,1]$ in two separate manufacturing sectors indexed by $J \in \{N, T\}$, where N represents nontradables and T tradables. For prices in these two sectors we introduce a slightly different index $\tilde{J} \in \{N, TH\}$, because the index T for prices is reserved for a different goods aggregate produced by distributors (see below). Manufacturers buy labor inputs from unions and capital inputs from investment goods producers. Sector N and T manufacturers sell to domestic distributors, and sector T manufacturers also sell to import agents in foreign countries, who in turn sell to distributors in those countries.[10] Manufacturers are perfectly competitive in their input markets and monopolistically competitive in the market for their output. Their price setting is subject to nominal rigidities. We first analyze the demands for their output, then turn to their technology, and finally describe their optimization problem.

Demands for manufacturers' output varieties are given by

$$Y_t^J(z) = \left(\int_0^1 Y_t^J(z,i)^{\frac{\sigma_J - 1}{\sigma_J}} di \right)^{\frac{\sigma_J}{\sigma_J - 1}} \quad , \quad Y_t^{TX}(1,j,z) = \left(\int_0^1 Y_t^{TX}(1,j,z,i)^{\frac{\sigma_J - 1}{\sigma_J}} di \right)^{\frac{\sigma_J}{\sigma_J - 1}} , \tag{34}$$

where $Y_t^J(z,i)$ and $Y_t^J(z)$ are variety i and total demands from domestic distributor z in sector J, and $Y_t^{TX}(1,j,z,i)$ and $Y_t^{TX}(1,j,z)$ are variety i and total demands for exports from country 1 to import agent z in country j. Cost minimization by distributors and import agents generates demands for varieties

$$Y_t^J(z,i) = \left(\frac{P_t^{\tilde{J}}(i)}{P_t^{\tilde{J}}} \right)^{-\sigma_J} Y_t^J(z) \quad , \quad Y_t^{TX}(1,j,z,i) = \left(\frac{P_t^{TH}(i)}{P_t^{TH}} \right)^{-\sigma_J} Y_t^{TX}(1,j,z) \, , \tag{35}$$

with price indices defined as

$$P_t^{\tilde{J}} = \left(\int_0^1 P_t^{\tilde{J}}(i)^{1-\sigma_J} di \right)^{\frac{1}{1-\sigma_J}} . \tag{36}$$

[10] There are also some small sales of aggregate manufacturing output back to manufacturing firms, related to manufacturers' need for resources to pay for adjustment costs.

The aggregate demand for variety i produced by sector J can be derived by simply integrating over all distributors, import agents and all other sources of manufacturing output demand. We obtain

$$Z_t^J(i) = \left(\frac{P_t^{\tilde{J}}(i)}{P_t^{\tilde{J}}} \right)^{-\sigma_J} Z_t^J \ , \tag{37}$$

where $Z_t^J(i)$ and Z_t^J remain to be specified by way of market clearing conditions for manufacturing goods.

The **technology** of each manufacturing firm differs depending on whether the raw materials sector is included. If it is included, the technology is given by a CES production function in capital $K_{t-1}^J(i)$, union labor $U_t^J(i)$ and raw materials $X_t^J(i)$, with elasticities of substitution ξ_{ZJ} between capital and labor, and ξ_{XJ} between raw materials and capital/labor. An adjustment cost $G_{X,t}^J(i)$ makes fast changes in raw materials inputs costly. Labor augmenting productivity is $T_t A_t^J$, where A_t^J is a country specific technology shock:[11,12]

$$Z_t^J(i) = F(K_{t-1}^J(i), U_t^J(i), X_t^J(i)) \tag{38}$$

$$= \mathfrak{T} \left(\left(1 - \alpha_{J_t}^X\right)^{\frac{1}{\xi_{XJ}}} \left(M_t^J(i)\right)^{\frac{\xi_{XJ}-1}{\xi_{XJ}}} + \left(\alpha_{J_t}^X\right)^{\frac{1}{\xi_{XJ}}} \left(X_t^J(i)\left(1 - G_{X,t}^J(i)\right)\right)^{\frac{\xi_{XJ}-1}{\xi_{XJ}}} \right)^{\frac{\xi_{XJ}}{\xi_{XJ}-1}} ,$$

$$M_t^J(i) = \left(\left(1 - \alpha_J^U\right)^{\frac{1}{\xi_{ZJ}}} \left(K_{t-1}^J(i)\right)^{\frac{\xi_{ZJ}-1}{\xi_{ZJ}}} + \left(\alpha_J^U\right)^{\frac{1}{\xi_{ZJ}}} \left(T_t A_t^J U_t^J(i)\right)^{\frac{\xi_{ZJ}-1}{\xi_{ZJ}}} \right)^{\frac{\xi_{ZJ}}{\xi_{ZJ}-1}} .$$

If the raw materials sector is not included, the technology is given by a CES production function in capital $K_t^J(i)$ and union labor $U_t^J(i)$, with elasticity of substitution ξ_{ZJ} between capital and labor:

$$Z_t^J(i) = F(K_{t-1}^J(i), U_t^J(i)) \tag{39}$$

$$= \mathfrak{T} \left(\left(1 - \alpha_J^U\right)^{\frac{1}{\xi_{ZJ}}} \left(K_{t-1}^J(i)\right)^{\frac{\xi_{ZJ}-1}{\xi_{ZJ}}} + \left(\alpha_J^U\right)^{\frac{1}{\xi_{ZJ}}} \left(T_t A_t^J U_t^J(i)\right)^{\frac{\xi_{ZJ}-1}{\xi_{ZJ}}} \right)^{\frac{\xi_{ZJ}}{\xi_{ZJ}-1}} .$$

We will from now on mostly ignore the version without raw materials sector, for which the optimality conditions can be derived in the same fashion as below.

11 Note that, for the sake of clarity, we make a notational distinction between two types of elasticities of substitution. Elasticities between continua of goods varieties, which give rise to market and pricing power, are denoted by a σ subscripted by the respective sectorial indicator. Elasticities between factors of production, both in manufacturing and in final goods distribution, are denoted by a ξ subscripted by the respective sectorial indicator.

12 The factor $\mathcal{T}$ is a constant that can be set different from one to obtain different levels of GDP per capita across countries.

Manufacturing firms are subject to three (GIMF with Financial Accelerator) or four (GIMF without Financial Accelerator) types of adjustment costs. First, quadratic inflation adjustment costs $G^J_{P,t}(i)$ are real resource costs that represent a demand for the output of sector J. Following Ireland (2001) and Laxton and Pesenti (2003), they are quadratic in changes in the rate of inflation rather than in price levels, which is essential in order to generate realistic inflation dynamics. Compared to versions of the Calvo (1983) price setting assumption such adjustment costs have the advantage of greater analytical tractability. We have:

$$G^J_{P,t}(i) = \frac{\phi_{P^J}}{2} Z^J_t \left(\frac{\frac{P^{\tilde{J}}_t(i)}{P^{\tilde{J}}_{t-1}(i)}}{\frac{P^{\tilde{J}}_{t-1}}{P^{\tilde{J}}_{t-2}}} - 1 \right)^2 . \tag{40}$$

To allow a flexible choice of inflation adjustment costs we also allow for a version of Rotemberg (1982) sticky prices, whereby deviations of the actual inflation rate from the inflation target $\bar{\pi}_t$ are costly. These may sometimes be preferable when working with a fixed exchange rates model, where sticky inflation can give rise to strong cycles. These costs are given by[13]

$$G^J_{P,t}(i) = \frac{\phi_{P^J}}{2} Z^J_t \left(\frac{P^{\tilde{J}}_t(i)}{P^{\tilde{J}}_{t-1}(i)} - \bar{\pi}_t \right)^2 . \tag{41}$$

Second, adjustment costs on raw materials inputs are given by[14]

$$G^J_{X,t}(i) = \frac{\phi^J_X}{2} \left(\frac{(X^J_t(i)/(gn)) - X^J_{t-1}}{X^J_{t-1}} \right)^2 , \tag{42}$$

the term gn enters to ensure that adjustment costs are zero along the balanced growth path.

Third, adjustment costs on labor hiring are given by

$$G^J_{U,t}(i) = \frac{\phi_U}{2} U^J_t \left(\frac{(U^J_t(i)/n) - U^J_{t-1}(i)}{U^J_{t-1}(i)} \right)^2 . \tag{43}$$

These costs are somewhat less common in the business cycle literature, and are only included as an option that can be switched off by setting $\phi_U = 0$.

Fourth, when the Financial Accelerator is absent manufacturers accumulate capital inside the firm. In that case they are subject to quadratic investment adjustment costs $G_{I,t}(i)$:

$$G^J_{I,t}(i) = \frac{\phi_I}{2} I^J_t \left(\frac{(I^J_t(i)/(gn)) - I^J_{t-1}(i)}{I^J_{t-1}(i)} \right)^2 , \tag{44}$$

[13] In all other instances of nominal rigidities that follow, GIMF offers this as one option. It will however not be mentioned again in this document.

[14] Note that, unlike other adjustment costs, this expression treats lagged inputs as external. This has proved more useful than the alternatives in our applied work.

where the term I_t^J outside the brackets is a scaling factor. Again without the Financial Accelerator, the law of motion of capital is relevant for the manufacturer, and is described by

$$\bar{K}_t^J(i) = \left(1 - \delta_{K_t}^J\right) \bar{K}_{t-1}^J(i) + S_t^{inv} I_t^J(i) \;, \tag{45}$$

where $\delta_{K_t}^J$ represents the depreciation rate of capital and S_t^{inv} is a shock to investment productivity. We allow for shocks to the depreciation rate of capital:

$$\delta_{K_t}^J = \bar{\delta}_K^J + S_t^{nwkshk} \;. \tag{46}$$

The relationship between the aggregate physical stock of this capital $\bar{K}_t^J$ and capital used in manufacturing K_t^J is given by

$$K_t^J = \bar{K}_t^J \;, \tag{47}$$

or in normalized form

$$\check{K}_t^J = \overline{\check{K}}_t^J \;. \tag{48}$$

It is assumed that each firm pays out each period's after tax nominal net cash flow as dividends $D_t^J(i)$. It maximizes the expected present discounted value of dividends. The discount rate it applies in this maximization includes the parameter θ so as to equate the discount factor of firms $\theta/\check{r}_t$ with the pricing kernel for nonfinancial income streams of their owners, myopic households, which equals $\beta\theta E_t\left(\lambda_{a+1,t+1}/\lambda_{a,t}\right)$. This equality follows directly from OLG households' first order condition for government debt holdings $\lambda_{a,t} = \beta E_t\left(\lambda_{a+1,t+1}\frac{i_t}{\pi_{t+1}\left(1+\xi_t^b\right)}\right)$.

Pre-tax net cash flow equals nominal revenue $P_t^{\tilde{J}}(i)Z_t^J(i)$ minus nominal cash outflows. The latter include the wage bill $V_t U_t^J(i)$, where V_t is the aggregate wage rate charged by unions, spending on raw materials $P_t^X X_t^J(i)$, where P_t^X is the price of raw materials, and the cost of capital. The latter is different depending on whether we use the version of GIMF without and with a Financial Accelerator. Without a Financial Accelerator the manufacturer accumulates capital directly and has associated investment outlays of $P_t^I\left(I_t^J(i) + G_{I,t}^J(i)\right)$, where P_t^I is the price of investment goods, and where both investment spending $I_t^J(i)$ and adjustment costs $G_{I,t}^J(i)$ represent a demand for investment goods output Z_t^I. With a Financial Accelerator capital is accumulated by the entrepreneur sector, from whom the manufacturer rents capital in the usual way, at a cost of $R_{k,t}^J K_t^J(i)$, where $R_t^{K^J}$ is the nominal rental cost of capital in sector J, with the real cost denoted $r_{k,t}^J$. Other components of pre-tax cash flow are price adjustment costs $P_t^{\tilde{J}} G_{P,t}^J(i)$ that represent a demand for sectorial manufacturing output Z_t^J, labor adjustment costs $V_t G_{U,t}(i)$ that represent a demand for labor L_t, and a fixed cost $P_t^{\tilde{J}} T_t \omega^J$. The fixed resource cost arises as long as the firm chooses to produce positive output. Net output in sector J is therefore equal to $\max(0, Z_t^J(i) - T_t\omega^J)$. The fixed cost is calibrated to make the steady state shares of economic profits, labor and capital in GDP consistent with the data. This becomes necessary because the model counterpart of the aggregate income share of capital equals not only the return to capital but also the profits of monopolistically competitive firms. With several layers of such firms the profits share becomes significant, and the capital share parameter in the production function has to be reduced accordingly, unless fixed costs are assumed. More importantly, the introduction of an additional parameter determining fixed costs allows us to simultaneously calibrate not only capital income shares and depreciation rates but also the investment to GDP ratio. This would otherwise be impossible. We calibrate fixed costs by first noting that, in normalized form, steady state monopoly profits equal $\check{Z}_t^J/\sigma_J$. We denote by s_π the share of these profits that remain after fixed costs have been paid, and we will calibrate this parameter to obtain the desired investment to GDP ratio. We assume that s_π is identical across the industries where fixed costs arise. Then fixed costs in manufacturing are given by

$$\omega^J = \frac{\bar{Z}^J}{\sigma_J}(1 - s_\pi) \ . \tag{49}$$

In the version of GIMF without Financial Accelerator net cash flow does not equal economic profit because investment expenditure represents a cash outflow but not an expenditure. The cash flow subject to the capital income tax is the nominal return to capital net of depreciation $\left(R_{k,t}^J - \delta_{K_t}^J P_t q_t^J\right) K_{t-1}^J(i)$. For GIMF without Financial Accelerator the total after tax net cash flow or dividend of the firm is[15]

$$\begin{aligned} D_t^J(i) &= P_t^{\tilde{J}}(i) Z_t^J(i) - V_t U_t^J(i) - P_t^X X_t^J(i) - P_t^I I_t^J(i) - P_t^{\tilde{J}} T_t \omega^J \\ &\quad - V_t G_{U,t}^J(i) - P_t^I G_{I,t}^J(i) - P_t^{\tilde{J}} G_{P,t}^J(i) - \tau_{k,t}\left[R_{k,t}^J - \delta_{K_t}^J P_t q_t^J\right] K_{t-1}^J(i) \ . \end{aligned} \tag{50}$$

For GIMF with Financial Accelerator the corresponding expression is

$$\begin{aligned} D_t^J(i) &= P_t^{\tilde{J}}(i) Z_t^J(i) - V_t U_t^J(i) - P_t^X X_t^J(i) - R_{k,t}^J K_{t-1}^J(i) \\ &\quad - V_t G_{U,t}^J(i) - P_t^{\tilde{J}} G_{P,t}^J(i) - P_t^{\tilde{J}} T_t \omega^J \ . \end{aligned} \tag{51}$$

[15] Note that the last term assumes that the depreciation allowance for capital income taxation purposes is evaluated at current market prices of installed capital $P_t q_t^J K_t^J$, as opposed to the book value of installed capital. While this may not correspond exactly to most real world tax systems, it does correspond exactly to the nominal economic loss to the firm due to capital depreciation.

The **optimization problem** of each manufacturing firm is (there is no optimization w.r.t. I_t^J for GIMF with Financial Accelerator)

$$\underset{\left\{P_{t+s}^{\tilde{J}}(i),U_{t+s}^J(i),I_{t+s}^J(i),K_{t+s}^J(i)\right\}_{s=0}^{\infty}}{Max} E_t\Sigma_{s=0}^{\infty}\tilde{R}_{t,s}D_{t+s}^J(i) \ , \tag{52}$$

subject to the definition of dividends (50) or (51), demands (37), production functions (38), and adjustment costs (40), (43) and, if applicable, (44). The first-order conditions for this problem are derived in some detail in Appendix D for GIMF without Financial Accelerator and in Appendix E for GIMF with Financial Accelerator. A key step is to recognize that all firms behave identically in equilibrium, so that $P_t^{\tilde{J}}(i) = P_t^{\tilde{J}}$ and $Z_t^J(i) = Z_t^J$. Let λ_t^J denote the real marginal cost of producing an additional unit of manufacturing output. Also, rescale the optimality conditions by technology and population as discussed above. Then the condition for $P_t^{\tilde{J}}(i)$ under sticky inflation is

$$\begin{aligned}\left[\frac{\sigma_J}{\sigma_J-1}\frac{\lambda_t^J}{p_t^{\tilde{J}}}-1\right] &= \frac{\phi_{P^J}}{\sigma_J-1}\left(\frac{\pi_t^{\tilde{J}}}{\pi_{t-1}^{\tilde{J}}}\right)\left(\frac{\pi_t^{\tilde{J}}}{\pi_{t-1}^{\tilde{J}}}-1\right) \\ &-E_t\frac{\theta gn}{\check{r}_{t+1}}\frac{\phi_{P^J}}{\sigma_J-1}\frac{p_{t+1}^{\tilde{J}}}{p_t^{\tilde{J}}}\frac{\check{Z}_{t+1}^J}{\check{Z}_t^J}\left(\frac{\pi_{t+1}^{\tilde{J}}}{\pi_t^{\tilde{J}}}\right)\left(\frac{\pi_{t+1}^{\tilde{J}}}{\pi_t^{\tilde{J}}}-1\right) \ ,\end{aligned} \tag{53}$$

while under sticky prices we have

$$\begin{aligned}\left[\frac{\sigma_J}{\sigma_J-1}\frac{\lambda_t^J}{p_t^{\tilde{J}}}-1\right] &= \frac{\phi_{P^J}}{\sigma_J-1}\pi_t^{\tilde{J}}\left(\pi_t^{\tilde{J}}-\bar{\pi}_t\right) \\ &-E_t\frac{\theta gn}{\check{r}_{t+1}}\frac{\phi_{P^J}}{\sigma_J-1}\frac{p_{t+1}^{\tilde{J}}}{p_t^{\tilde{J}}}\frac{\check{Z}_{t+1}^J}{\check{Z}_t^J}\pi_{t+1}^{\tilde{J}}\left(\pi_{t+1}^{\tilde{J}}-\bar{\pi}_t\right) \ .\end{aligned} \tag{54}$$

The first order condition for labor demand $U_t^J(i)$ is

$$\left(\frac{\lambda_t^J}{\check{v}_t}\check{F}_{U,t}^J-1\right) = \phi_U\left(\frac{\check{U}_t}{\check{U}_{t-1}}\right)\left(\frac{\check{U}_t-\check{U}_{t-1}}{\check{U}_{t-1}}\right)-\frac{\theta gn}{\check{r}_{t+1}}\phi_U\frac{\check{v}_{t+1}}{\check{v}_t}\left(\frac{\check{U}_{t+1}}{\check{U}_t}\right)^2\left(\frac{\check{U}_{t+1}-\check{U}_t}{\check{U}_t}\right) \ , \tag{55}$$

where $\check{F}_{U,t}^J$ is the marginal product of labor

$$\check{F}_{U,t}^J = \mathcal{T}\left(\frac{\left(1-\alpha_{J_t}^X\right)\check{Z}_t^J}{\mathcal{T}\check{M}_t^J}\right)^{\frac{1}{\xi_{XJ}}} A_t^J\left(\frac{\alpha_J^U\check{M}_t^J}{A_t^J\check{U}_t^J}\right)^{\frac{1}{\xi_{ZJ}}} \ . \tag{56}$$

The first order condition for raw materials demand $X_t^J(i)$ is

$$p_t^X = \lambda_t^J\check{F}_{X,t}^J \ , \tag{57}$$

where $\check{F}^J_{X,t}$ is the marginal product of raw materials

$$\check{F}^J_{X,t} = \mathcal{T} \left(\frac{\alpha^X_{J_t} \check{Z}^J_t}{\mathcal{T} \check{X}^J_t \left(1 - G^J_{X,t}\right)} \right)^{\frac{1}{\xi_{XJ}}} \left(1 - G^J_{X,t} - \phi^J_X \frac{\check{X}^J_t}{\check{X}^J_{t-1}} \left(\frac{\check{X}^J_t - \check{X}^J_{t-1}}{\check{X}^J_{t-1}} \right) \right) \ . \tag{58}$$

For GIMF without a Financial Accelerator there is no equivalent condition determining the real return to capital $r^J_{k,t}$, because capital is owned by the firm and not rented through a market. However, in order to determine the profits and capital income taxes payable to them, the fiscal authorities must impute $r^J_{k,t}$. We assume that it is imputed to be equivalent to what would be obtained if capital was rented through a market, and which would obtain also in the version with a Financial Accelerator, namely

$$r^J_{k,t} = \lambda^J_t \check{F}^J_{K,t} \ , \tag{59}$$

where $\check{F}^J_{K,t}$ is the marginal product of capital

$$\check{F}^J_{K,t} = \mathcal{T} \left(\frac{\left(1 - \alpha^X_{J_t}\right) \check{Z}^J_t}{\mathcal{T} \check{M}^J_t} \right)^{\frac{1}{\xi_{XJ}}} \left(\frac{\left(1 - \alpha^U_J\right) \check{M}^J_t}{\left(\check{K}^J_{t-1} / (gn)\right)} \right)^{\frac{1}{\xi_{ZJ}}} \ . \tag{60}$$

For the sake of completeness we add here the marginal products of labor and capital for the version of GIMF without raw materials. They are

$$\check{F}^J_{U,t} = \mathcal{T} A^J_t \left(\frac{\alpha^U_J \check{Z}^J_t}{A^J_t \check{U}^J_t} \right)^{\frac{1}{\xi_{ZJ}}} \ , \tag{61}$$

$$\check{F}^J_{K,t} = \mathcal{T} \left(\frac{\left(1 - \alpha^U_J\right) \check{Z}^J_t}{\left(\check{K}^J_{t-1} / (gn)\right)} \right)^{\frac{1}{\xi_{ZJ}}} \ . \tag{62}$$

For the version without the Financial Accelerator the investment and capital decisions take place in the manufacturing firm. In that case the first order condition for investment demand $I^J_t(i)$ is

$$q^J_t S^{inv}_t = p^I_t + \phi_I p^I_t \left(\frac{\check{I}^J_t}{\check{I}^J_{t-1}} \right) \left(\frac{\check{I}^J_t - \check{I}^J_{t-1}}{\check{I}^J_{t-1}} \right) - E_t \frac{\theta gn}{\check{r}_{t+1}} \phi_I p^I_{t+1} \left(\frac{\check{I}^J_{t+1}}{\check{I}^J_t} \right)^2 \left(\frac{\check{I}^J_{t+1} - \check{I}^J_t}{\check{I}^J_t} \right) \ , \tag{63}$$

while the Euler equation for capital, i.e. the first order condition with respect to $K^J_t(i)$, is[16]

$$q^J_t = \frac{\theta}{\check{r}_{t+1}} E_t \left[q^J_{t+1} (1 - \delta^J_{K_{t+1}}) + r^J_{k,t+1} - \tau_{k,t+1} \left(r^J_{k,t+1} - \delta^J_{K_{t+1}} q^J_{t+1} \right) \right] \ . \tag{64}$$

[16] The optimization setup uses the identity (47).

Without Financial Accelerator the rescaled aggregate dividends of firms in each sector are

$$\check{d}_t^J = \left[p_t^{\tilde{J}} \check{Z}_t^J - \check{v}_t \check{U}_t^J - p_t^X \check{X}_t^J - p_t^I \check{I}_t^J - \check{v}_t \check{G}_{U,t}^J - p_t^I \check{G}_{I,t}^J - p_t^{\tilde{J}} \check{G}_{P,t}^J - p_t^{\tilde{J}} \omega^J\right] \\ -\tau_{k,t} \left[r_{k,t}^J - \delta_{K_t}^J q_t^J\right] \left(\check{K}_{t-1}^J / (gn)\right) . \quad (65)$$

With the Financial Accelerator they are

$$\check{d}_t^J = \left[p_t^{\tilde{J}} \check{Z}_t^J - \check{v}_t \check{U}_t^J - p_t^X \check{X}_t^J - r_{k,t}^J \left(\check{K}_{t-1}^J / (gn)\right) - \check{v}_t \check{G}_{U,t}^J - p_t^{\tilde{J}} \check{G}_{P,t}^J - p_t^{\tilde{J}} \omega^J\right] . \quad (66)$$

We define aggregate capital and investment as

$$\check{I}_t = \check{I}_t^N + \check{I}_t^T , \quad (67)$$

$$\check{K}_t = \check{K}_t^N + \check{K}_t^T . \quad (68)$$

Finally, we turn to the **market clearing** conditions for nontradables and tradables. For GIMF without Financial Accelerator they equate the output of each sector to the demands of distributors, of manufacturers themselves for fixed and adjustment costs, and in the case of tradables to the demands of foreign import agents:[17]

$$\check{Z}_t^N = \check{Y}_t^N + \omega^N + \check{G}_{P,t}^N . \quad (69)$$

$$\check{Z}_t^T(1) = \check{Y}_t^{TH}(1) + \omega^T(1) + \check{G}_{P,t}^T(1) + \tilde{p}_t^{exp} \Sigma_{j=2}^{\tilde{N}} \check{Y}_t^{TX}(1, j) . \quad (70)$$

The term $\tilde{p}_t^{exp}$ in the second market clearing condition refers to unit root shocks to the relative price of exported goods. Specifically, tradables output is converted to exports $\check{Y}_t^{TX}$ using a technology that multiplies tradables output by $T_t^{exp} = 1/\tilde{p}_t^{exp}$, where $\tilde{p}_t^{exp}$ is a unit root shock with zero trend growth. For GIMF with a Financial Accelerator these conditions have to be augmented by the net effects of entrepreneurs' output destroying net worth shocks $\check{S}_t^{J,nwyshk}$, and their real resource costs due to bankruptcies and capital utilization $r\check{c}_t^J$. We have

$$\check{Z}_t^N = \check{Y}_t^N + \omega^N + \check{G}_{P,t}^N + \check{S}_t^{N,nwyshk} + r\check{c}_t^N . \quad (71)$$

$$\check{Z}_t^T(1) = \check{Y}_t^{TH}(1) + \omega^T(1) + \check{G}_{P,t}^T(1) + \tilde{p}_t^{exp} \Sigma_{j=2}^{\tilde{N}} \check{Y}_t^{TX}(1, j) + r\check{c}_t^T(1) + \check{S}_t^{T,nwyshk}(1) . \quad (72)$$

[17] The tradables market clearing condition is reported for the example of country 1.

6 Capital Producers

These agents produce the capital stock used by entrepreneurs in the nontradables and tradables sectors, indexed as before by $J \in \{N, T\}$. They are competitive price takers. Capital producers are owned by households, who receive their dividends as lump-sum transfers. They purchase previously installed capital $\tilde{K}^J_{t-1}$ from entrepreneurs and investment goods I^J_t from investment goods producers to produce new installed capital $\tilde{K}^J_t$ according to

$$\tilde{K}^J_t = \tilde{K}^J_{t-1} + S^{inv}_t I^J_t \ , \tag{73}$$

where S^{inv}_t is an investment demand shock. They are subject to investment adjustment costs

$$G^J_{I,t} = \frac{\phi_I}{2} I^J_t \left(\frac{(I^J_t/(gn)) - I^J_{t-1}}{I^J_{t-1}} \right)^2 \ . \tag{74}$$

The nominal price level of previously installed capital is denoted by Q^J_t. Since the marginal rate of transformation from previously installed to newly installed capital is one, the price of new capital is also Q^J_t. The optimization problem is to maximize the present discounted value of dividends by choosing the level of new investment I^J_t:[18]

$$\underset{\{I^J_{t+s}\}_{s=0}^{\infty}}{Max} \ E_t \Sigma_{s=0}^{\infty} \tilde{R}_{t,s} D^{K^J}_{t+s} \ , \tag{75}$$

$$D^{K^J}_t = Q^J_t \left(\tilde{K}^J_{t-1} + S^{inv}_t I^J_t \right) - Q^J_t \tilde{K}^J_{t-1} - P^I_t \left(I^J_t + G^J_{I,t} \right) \ . \tag{76}$$

The solution to this problem is

$$q^J_t S^{inv}_t = p^I_t + \phi_I p^I_t \left(\frac{\check{I}^J_t}{\check{I}^J_{t-1}} \right) \left(\frac{\check{I}^J_t - \check{I}^J_{t-1}}{\check{I}^J_{t-1}} \right) - E_t \frac{\theta gn}{\check{r}_{t+1}} \phi_I p^I_{t+1} \left(\frac{\check{I}^J_{t+1}}{\check{I}^J_t} \right)^2 \left(\frac{\check{I}^J_{t+1} - \check{I}^J_t}{\check{I}^J_t} \right) \ . \tag{77}$$

The stock of physical capital evolves as

$$\bar{K}^J_t = \left(1 - \delta^J_{K_t}\right) \bar{K}^J_{t-1} + S^{inv}_t I^J_t \ . \tag{78}$$

As before, we allow for shocks to the deprecation rate of capital, which in the context of the Financial Accelerator we will refer to as capital destroying net worth shocks:

$$\delta^J_{K_t} = \bar{\delta}^J_K + S^{nwkshk}_t \ . \tag{79}$$

[18] Any value of capital if profit maximizing.

Physical capital $\bar{K}_t^J$ is different from the capital rented by manufacturers K_t^J because the stock of physical capital is subject to variable capital utilization u_t^J. The normalized relationship between physical capital $\bar{K}^J$ and capital used in manufacturing K^J is therefore given by

$$\check{K}_t^J = u_t^J \overline{\check{K}}_t^J \ . \tag{80}$$

The real value of dividends is given by

$$\check{d}_t^{K^J} = q_t^J S_t^{inv} \check{I}_t^J - p_t^I \left(\check{I}_t^J + \check{G}_{I,t}^J \right) \ . \tag{81}$$

We let $\check{d}_t^K = \check{d}_t^{K^N} + \check{d}_t^{K^T}$, and also $\check{I}_t = \check{I}_t^N + \check{I}_t^T$, $\bar{K}_t = \bar{K}_t^N + \bar{K}_t^T$.

7 Entrepreneurs and Banks

Entrepreneurs in sectors $J \in \{N, T\}$ purchase a capital stock from capital producers and rent it to manufacturers. Each entrepreneur j finances his time t capital holdings (at current market prices) $Q_t^J \bar{K}_t^J(j)$ with a combination of his net worth $N_t^J(j)$ and a bank loan $B_t^J(j)$. His balance sheet constraint is therefore given by

$$Q_t^J \bar{K}_t^J(j) = N_t^J(j) + B_t^J(j) \ , \tag{82}$$

or in real normalized terms by

$$q_t^J \overline{\check{K}}_t^J(j) = \check{n}_t^J(j) + \check{b}_t^J(j) \ . \tag{83}$$

After the capital purchase each entrepreneur draws an **idiosyncratic** shock which changes $\bar{K}_t^J(j)$ to $\omega_{t+1}^J \bar{K}_t^J(j)$ at the beginning of period $t+1$, where ω_{t+1}^J is a unit mean lognormal random variable distributed independently over time and across entrepreneurs. The standard deviation of $\ln(\omega_{t+1}^J)$, σ_{t+1}^J, is itself a stochastic process. While the realization of ω_{t+1}^J is not known at the time the entrepreneur makes his capital decision, the value of σ_{t+1}^J is known. The cumulative distribution function of ω_{t+1}^J is given by $\Pr(\omega_{t+1}^J \leq x) = F_{t+1}^J(x)$.

After observing the time t **aggregate** shocks the entrepreneur decides on the time t level of capital utilization u_t^J, and then rents out capital services $K_t^J(j) = u_t^J \bar{K}_t^J(j)$. High capital utilization gives rise to high costs in terms of sector J goods, according to the convex function $a(u_t^J)\omega_t^J \bar{K}_{t-1}^J(j)$, where we specify the adjustment cost function as[19]

[19] This follows Christiano, Motto and Rostagno (2007), "Financial Factors in Business Cycles". Papers where the model is linearized prior to solving it only require the elasticity σ_a of the function $a(u_t)$. Because GIMF is solved in nonlinear form we require a full functional form.

$$a(u_t^J) = \frac{1}{2}\phi_a^J \sigma_a^J \left(u_t^J\right)^2 + \phi_a^J \left(1 - \sigma_a^J\right) u_t^J + \phi_a^J \left(\frac{\sigma_a^J}{2} - 1\right) . \tag{84}$$

The entrepreneur chooses u_t^J to solve

$$\underset{u_t^J}{Max} \quad \left[u_t^J r_{k,t}^J - a(u_t^J)\right] \left(1 - \tau_{k,t}\right) \omega_t^J \bar{K}_{t-1}^J(j) , \tag{85}$$

which has the solution

$$r_{k,t}^J = \phi_a^J \sigma_a^J u_t^J + \phi_a^J \left(1 - \sigma_a^J\right) . \tag{86}$$

The entrepreneur's real ex-post, after tax return to utilized capital is given by

$$ret_{k,t}^J = \frac{\left(u_t^J r_{k,t}^J - a(u_t^J) + \left(1 - \delta_{K_t}^J\right) q_t^J\right) - \tau_{k,t} \left(u_t^J r_{k,t}^J - a(u_t^J) - \delta_{K_t}^J q_t^J\right)}{q_{t-1}^J} . \tag{87}$$

We assume that the entrepreneur receives a standard debt contract from the bank. This specifies a loan amount B_t^J and a gross rate of interest $i_{B,t+1}^J$ to be paid if ω_{t+1}^J is high enough. Entrepreneurs who draw ω_{t+1}^J below a cutoff level $\bar{\omega}_{t+1}^J$ cannot pay this interest rate and go bankrupt. They must hand over everything they have to the bank, but the bank can only recover a time-varying fraction $(1 - \mu_{t+1}^J)$ of the value of such firms. The cutoff $\bar{\omega}_{t+1}^J$ is defined as follows:

$$\bar{\omega}_{t+1}^J ret_{k,t+1}^J Q_t^J \bar{K}_t^J(j) = i_{B,t+1}^J B_t^J(j) , \tag{88}$$

where $ret_{k,t+1}^J$ is the nominal ex-post after tax return to utilized capital. The bank finances its loans to entrepreneurs by borrowing from households. We assume that the bank pays households a nominal rate of return $\check{i}_t = i_t / \left(1 + \xi_t^b\right)$ that is not contingent on the realization of time $t + 1$ shocks. The parameters of the entrepreneur's debt contract are chosen to maximize entrepreneurial utility, subject to zero profits in each state of nature for the bank and to the requirement that $\check{i}_t$ be non-contingent on time $t + 1$ shocks. This implies that $i_{B,t+1}^J$ and $\bar{\omega}_{t+1}^J$ are both functions of time $t + 1$ aggregate shocks.

The bank's zero profit or participation constraint is given by:[20]

$$\left(1 - F(\bar{\omega}_{t+1}^J)\right) i_{B,t+1}^J B_t^J(j) + \left(1 - \mu_{t+1}^J\right) \int_0^{\bar{\omega}_{t+1}^J} Q_t^J \bar{K}_t^J(j) ret_{k,t+1}^J \omega f(\omega) d\omega = \check{i}_t B_t^J(j) . \tag{89}$$

This states that the stochastic payoff to lending on the l.h.s. must equal the non-stochastic payment to depositors on the r.h.s. in each state of nature. The first term on the l.h.s. is the nominal interest

[20] Note the absence of expectations operators because this equation has to hold in each state of nature. Likewise for subsequent equations.

income on loans for borrowers whose idiosyncratic shock exceeds the cutoff level, $\omega^J_{t+1} \geq \bar{\omega}^J_{t+1}$. The second term is the amount collected by the bank in case of the borrower's bankruptcy, where $\omega^J_{t+1} < \bar{\omega}^J_{t+1}$. This cash flow is based on the return $ret^J_{k,t+1}\omega$ on capital investment $Q^J_t \bar{K}^J_t(j)$, but multiplied by the factor $\left(1-\mu^J_{t+1}\right)$ to reflect a proportional bankruptcy cost μ^J_{t+1}. Next we rewrite (89) by using (88) and (82):

$$\begin{aligned} & \left[\left(1-F(\bar{\omega}^J_{t+1})\right)\bar{\omega}^J_{t+1} + \left(1-\mu^J_{t+1}\right)\int_0^{\bar{\omega}^J_{t+1}} \omega f(\omega) d\omega\right] ret^J_{k,t+1} Q^J_t \bar{K}^J_t(j) \\ = & \ \check{i}_t Q^J_t \bar{K}^J_t(j) - \check{i}_t N^J_t(j) \ . \end{aligned} \tag{90}$$

We adopt a number of definitions that simplify the following derivations. First, note that capital earnings are given by $ret^J_{k,t+1} Q^J_t \bar{K}^J_t(j)$. The lender's gross share in capital earnings is then defined as

$$\Gamma(\bar{\omega}^J_{t+1}) \equiv \int_0^{\bar{\omega}^J_{t+1}} \omega^J_{t+1} f(\omega^J_{t+1}) d\omega^J_{t+1} + \bar{\omega}^J_{t+1} \int_{\bar{\omega}^J_{t+1}}^{\infty} f(\omega^J_{t+1}) d\omega^J_{t+1} \ , \tag{91}$$

while his monitoring costs share in capital earnings is given by $\mu^J_{t+1} G(\bar{\omega}^J_{t+1})$, where

$$G(\bar{\omega}^J_{t+1}) = \int_0^{\bar{\omega}^J_{t+1}} \omega^J_{t+1} f(\omega^J_{t+1}) d\omega^J_{t+1} \ . \tag{92}$$

The lender's net share in capital earnings is therefore $\Gamma(\bar{\omega}^J_{t+1}) - \mu^J_{t+1} G(\bar{\omega}^J_{t+1})$. The entrepreneur's share in capital earnings on the other hand is given by

$$1-\Gamma(\bar{\omega}^J_{t+1}) = \int_{\bar{\omega}^J_{t+1}}^{\infty} \left(\omega^J_{t+1} - \bar{\omega}^J_{t+1}\right) f(\omega^J_{t+1}) d\omega^J_{t+1} \ . \tag{93}$$

Using this notation and denoting the multiplier of the participation constraint by λ_t, the entrepreneur's optimization problem can be written as

$$\begin{aligned} & \underset{\bar{K}^J_t(j), \bar{\omega}^J_{t+1}}{Max} \ \left(1-\Gamma(\bar{\omega}^J_{t+1})\right) ret^J_{k,t+1} Q^J_t \bar{K}^J_t(j) \\ & +\lambda_t \left\{ \left(\Gamma(\bar{\omega}^J_{t+1}) - \mu^J_{t+1} G(\bar{\omega}^J_{t+1})\right) ret^J_{k,t+1} Q^J_t \bar{K}^J_t(j) - \check{i}_t Q^J_t \bar{K}^J_t(j) + \check{i}_t N^J_t(j) \right\} \ . \end{aligned} \tag{94}$$

Before deriving the optimality conditions we rewrite this expression by dividing through by $\check{i}_t N^J_t(j)$, rewriting the resulting expression in terms of normalized variables, and finally replacing nominal returns by real returns:

$$\underset{\overline{\breve{K}}_t^J(j),\bar{\omega}_{t+1}^J}{Max} \left(1-\Gamma(\bar{\omega}_{t+1}^J)\right)\frac{r\breve{e}t_{k,t+1}^J}{\breve{r}_{t+1}}\frac{q_t^J\overline{\breve{K}}_t^J(j)}{\breve{n}_t^J(j)} \tag{95}$$
$$+\lambda_t\left\{\left(\Gamma(\bar{\omega}_{t+1}^J)-\mu_{t+1}^J G(\bar{\omega}_{t+1}^J)\right)\frac{r\breve{e}t_{k,t+1}^J}{\breve{r}_{t+1}}\frac{q_t^J\overline{\breve{K}}_t^J(j)}{\breve{n}_t^J(j)}-\frac{q_t^J\overline{\breve{K}}_t^J(j)}{\breve{n}_t^J(j)}+1\right\}.$$

We let $\Gamma_{t+1}^J = \Gamma(\bar{\omega}_{t+1}^J)$, $G_{t+1}^J = G(\bar{\omega}_{t+1}^J)$, $\Gamma'_{J,t+1} = \partial\Gamma_{t+1}^J/\partial\bar{\omega}_{t+1}^J$ and $G'_{J,t+1} = \partial G_{t+1}^J/\partial\bar{\omega}_{t+1}^J$. We obtain the following first-order condition with respect to $\bar{\omega}_{t+1}^J$:

$$-\Gamma'_{J,t+1}\frac{r\breve{e}t_{k,t+1}^J}{\breve{r}_{t+1}}\frac{q_t^J\overline{\breve{K}}_t^J(j)}{\breve{n}_t^J(j)}+\lambda_t\left\{\left(\Gamma'_{J,t+1}-\mu_{t+1}^J G'_{J,t+1}\right)\frac{r\breve{e}t_{k,t+1}^J}{\breve{r}_{t+1}}\frac{q_t^J\overline{\breve{K}}_t^J(j)}{\breve{n}_t^J(j)}\right\}=0\ , \tag{96}$$

which implies

$$\lambda_t = \frac{\Gamma'_{J,t+1}}{\Gamma'_{J,t+1}-\mu_{t+1}^J G'_{J,t+1}}\ . \tag{97}$$

The condition for the **optimal loan contract**, that is the first-order condition with respect to $\overline{\breve{K}}_t^J(j)$, can be written using (97) as[21]

$$\left(1-\Gamma_t^J\right)\frac{r\breve{e}t_{k,t}^J}{\breve{r}_t}+\frac{\Gamma'_{J,t}}{\Gamma'_{J,t}-\mu_t^J G'_{J,t}}\left\{\frac{r\breve{e}t_{k,t}^J}{\breve{r}_t}\left(\Gamma_t^J-\mu_t^J G_t^J\right)-1\right\}=0\ , \tag{98}$$

where we have replaced time $t+1$ subscripts with time t subscripts everywhere because this condition has to hold for each state of nature, that is it has to hold exactly ex-post. The normalized **lender's zero profit condition** is

$$\frac{q_t^J\overline{\breve{K}}_t^J}{\breve{n}_t^J}\frac{r\breve{e}t_{k,t+1}^J}{\breve{r}_{t+1}}\left(\Gamma_{t+1}^J-\mu_{t+1}^J G_{t+1}^J)\right)-\frac{q_t^J\overline{\breve{K}}_t^J}{\breve{n}_t^J}+1=0\ . \tag{99}$$

Notice that we have omitted entrepreneur specific indices j for capital and net worth and replaced them with the corresponding aggregate variables. This is because each entrepreneur faces the same returns $r\breve{e}t_{k,t+1}^J$ and $\breve{r}_{t+1}$, and the same risk environment characterizing the functions Γ and G. Aggregation of the model over entrepreneurs is then trivial because both borrowing and capital purchases are proportional to the entrepreneur's level of net worth.

A key problem for coding the Financial Accelerator version of GIMF in a standard software such as TROLL and DYNARE consists of finding a closed form representation for the terms Γ_t^J, G_t^J and

[21] Note that this condition has to hold for each state of nature and at all times. When coding GIMF it has to be coded for time t rather than time $t+1$.

their derivatives. In TROLL we can use the hard-wired (like e.g. LOG) PNORM function, which is the c.d.f. of the standard normal distribution. In Appendix F we therefore derive the relevant expressions in terms of PNORM, for which we use the notation $\Phi(.)$. We obtain the following set of equations, starting with an auxiliary variable $\bar{z}_t^J$:

$$\bar{z}_t^J = \frac{\ln(\bar{\omega}_t^J) + \frac{1}{2}\left(\sigma_t^J\right)^2}{\sigma_t^J} \ , \tag{100}$$

$$f\left(\bar{\omega}_t^J\right) = \frac{1}{\sqrt{2\pi}\bar{\omega}_t^J \sigma_t^J} \exp\left\{-\frac{1}{2}\left(\bar{z}_t^J\right)^2\right\} \ , \tag{101}$$

$$\Gamma_t^J = \Phi\left(\bar{z}_t^J - \sigma_t^J\right) + \bar{\omega}_t^J\left(1 - \Phi\left(\bar{z}_t^J\right)\right) \ , \tag{102}$$

$$G_t^J = \Phi\left(\bar{z}_t^J - \sigma_t^J\right) \ , \tag{103}$$

$$\Gamma'_{J,t} = 1 - \Phi\left(\bar{z}_t^J\right) \ , \tag{104}$$

$$G'_{J,t} = \bar{\omega}_t^J f\left(\bar{\omega}_t^J\right) \ . \tag{105}$$

As for the evolution of entrepreneurial net worth, we first note that banks make zero profits at all times. The difference between the aggregate returns to capital net of bankruptcy costs and the sum of deposit interest paid by banks to households therefore goes entirely to entrepreneurs and accumulates. To rule out a situation where over time so much net worth accumulates that entrepreneurs no longer need any loans, we assume that they regularly pay out to households dividends which, in terms of sector J output, are given by div_t^J. Net worth is also subject to output destroying shocks $S_t^{J,nwyshk}$. We assume that for an individual entrepreneur both dividends and output destroying shocks are proportional to his net worth, which given our above result concerning the proportionality of borrowing and capital purchases to net worth implies that the evolution of aggregate net worth is a straightforward aggregation of the evolution of entrepreneur specific net worth. Nominal aggregate net worth therefore evolves as

$$N_t^J = ret_{k,t}^J Q_{t-1}^J \overline{\check{K}}_{t-1}^J \left(1 - \mu_t^J G_t^J\right) - \check{i}_{t-1} B_{t-1}^J - P_t^{\tilde{J}}\left(div_t^J + S_t^{J,nwyshk}\right) \ . \tag{106}$$

This can be combined with the aggregate version of the balance sheet constraint (82) and normalized to yield

$$\check{n}_t^J = \frac{\check{r}_t}{gn}\check{n}_{t-1}^J + q_{t-1}^J\overline{\check{K}}_{t-1}^J\left(\frac{r\check{e}t_{k,t}^J}{gn}\left(1-\mu_t^J G_t^J\right) - \frac{\check{r}_t}{gn}\right) - p_t^{\tilde{J}}\left(d\check{i}v_t^J + \check{S}_t^{J,nwyshk}\right) \ . \tag{107}$$

Dividends in turn are given by the following expressions:

$$d\check{i}v_t^J = i\check{n}c_t^J + \theta_{nw}^J\left(\check{n}_t^J - \check{n}_t^{J,filt}\right) \tag{108}$$

$$p_t^{\tilde{J}}\ i\check{n}c_t^J = E_t\frac{S_t^{J,nwd}}{\left(k_h^{incJ} - k_l^{incJ} + 1\right)}\Sigma_{k=k_l^{incJ}}^{k_h^{incJ}}\left[\check{n}_{t+j}^J + p_{t+j}^{\tilde{J}}\left(d\check{i}v_{t+j}^J + \check{S}_{t+j}^{J,nwyshk}\right)\right] \ , \tag{109}$$

$$\check{n}_t^{J,filt} = E_t\Sigma_{k=k_l^{nw}}^{k_h^{nw}}\left(\check{n}_{t+j}^J\right)/\left(k_h^{nw} - k_l^{nw} + 1\right) \ . \tag{110}$$

Regular dividends, given by expression (109), are a fraction $S_t^{J,nwd}$ (with $\bar{S}^{J,nwd}$ typically in a range between 0 and 0.05) of smoothed (moving average) gross returns on net worth invested in the previous period, as per equation (107), with k_h^{incJ}/k_l^{incJ} the maximum lead/lag of the moving average. The dividend related net worth shock $S_t^{J,nwd}$ can cause temporary losses or gains of net worth that are a pure redistribution between households and entrepreneurs, without direct resource implications. The second determinant of dividends in (108) consists of a dividend response to deviations of net worth from its long-run value, the latter proxied by a moving average of past and future values of net worth. This allows us to model dividend policy as a tool to rebuild net worth more quickly following a negative shock. The parameter θ_{nw}^J (typically in a range between 0 and 0.05) measures the increase/decrease in dividends if net worth rises/falls below its long-run value. The relative price p_t^J enters because dividends are in units of sector J output while net worth is in units of final output. We define

$$\check{d}_t^{EP} = p_t^N d\check{i}v_t^N + p_t^{TH} d\check{i}v_t^T \ . \tag{111}$$

Output and capital destroying net worth shocks are easier to calibrate if they are expressed as fractions of steady state net worth.[22] We therefore adopt the definitions

$$\check{S}_t^{J,nwy} = \frac{p_t^{\tilde{J}}\check{S}_t^{J,nwyshk}}{\bar{n}^J} \ , \tag{112}$$

$$\check{S}_t^{J,nwk} = \frac{\check{S}_t^{J,nwkshk} q_t^J\left(\left(\overline{\check{K}}_{t-1}^J\right)/(gn)\right)}{\bar{n}^J} \ , \tag{113}$$

and express the shock processes as autocorrelated shocks to $\check{S}_t^{J,nwy}$ and $\check{S}_t^{J,nwk}$.

[22] Dividend related shocks are easier to calibrate as they are already in terms of a share of gross returns on net worth.

Finally, we define the sector J bankruptcy and capital utilization resource cost, which has to be paid out of the output of sector J, as

$$rc_t^J = \frac{\frac{\overline{K}_{t-1}^J}{gn}\left(r\check{e}t_{k,t}^J q_{t-1}^J \mu_t^J G_t^J + a(u_t^J)\right)}{p_t^J} \ . \tag{114}$$

8 Raw Materials Producers

In each period each country receives an endowment flow of raw materials $\check{X}_t^{sup}$ that, in the absence of exogenous shocks, is constant in normalized terms (i.e. it grows at the rate g). This endowment is sold to manufacturers worldwide, with total demand for each country given by $\check{X}_t^{dem}$. The value of a country's normalized raw materials exports is therefore given by

$$\check{X}_t^x = p_t^X(\check{X}_t^{sup} - \check{X}_t^{dem}). \tag{115}$$

The world market for raw materials is perfectly competitive, with flexible prices that are arbitraged worldwide. A constant share s_d^x of steady state (after normalization) raw materials revenue is paid out to domestic factors of production as dividends $\bar{d}^X$. The rest is divided in fixed shares $(1 - s_f^x)$ and $s_f^x = \Sigma_{j=2}^{\tilde{N}} s_f^x(1,j)$ between payments to the government $\check{g}_t^X$, for the case of publicly owned producers, and dividends to foreign owners in all other countries $\check{f}_t^X$. This means that all benefits of favorable raw materials price shocks accrue exclusively to the government and foreigners, and vice versa for unfavorable shocks. This corresponds more closely to the situation of many countries' raw materials sectors than the polar opposite assumption of assuming equal shares between the three recipients at all times. We have

$$\bar{d}^X = s_d^x \bar{p}^X \bar{X}^{sup} \ , \tag{116}$$

$$\check{f}_t^X(1,j) = s_f^x(1,j)\left(p_t^X \check{X}_t^{sup} - \bar{d}^X\right) \ , \tag{117}$$

$$\check{f}_t^X = \check{f}_t^X(1) = \Sigma_{j=2}^{\tilde{N}} \check{f}_t^X(1,j) \ , \tag{118}$$

$$\check{g}_t^X = p_t^X \check{X}_t^{sup} - \bar{d}^X - \check{f}_t^X \ , \tag{119}$$

where by international arbitrage we have

$$p_t^X = p_t^{X^*} e_t \ . \tag{120}$$

The dividends received by country 1 households from ownership of country j raw materials producers are then given by

$$\check{d}_t^F(1,j) = \check{f}_t^X(j,1)\frac{e_t(1)}{e_t(j)} \ , \tag{121}$$

and aggregate dividends are

$$\check{d}_t^F = \check{d}_t^F(1) = \Sigma_{j=2}^{\tilde{N}} \check{d}_t^F(1,j) \ . \tag{122}$$

The raw materials sector is subject to shocks to domestic supply $\check{X}_t^{sup}$ and to foreign demand, the latter via the raw materials share parameter in the manufacturing ($\alpha_{J_t}^X$) and retail ($\alpha_{C_t}^X$) sectors. Total demand for each country is given by

$$\check{X}_t^{dem} = \check{X}_t^T + \check{X}_t^N + \check{X}_t^C \ , \tag{123}$$

where $\check{X}_t^C$ is demand from the retail sector, that is directly from household consumption. The market clearing condition for the raw materials sector is worldwide, and given by

$$\Sigma_{j=1}^{\tilde{N}} \left(\check{X}_t^{sup(j)} - \check{X}_t^{dem(j)} \right) = 0 \ . \tag{124}$$

9 Unions

There is a continuum of unions indexed by $i \in [0,1]$. Unions buy labor from households and sell labor to manufacturers. They are perfectly competitive in their input market and monopolistically competitive in their output market. Their wage setting is subject to nominal rigidities. We first analyze the demands for union output and then describe their optimization problem.

Demand for unions' labor output varieties comes from manufacturing firms $z \in [0,1]$ in sectors $J \in \{N,T\}$. The demand for union labor by firm z in sector J is given by a CES production function with time-varying elasticity of substitution σ_{U_t},

$$U_t^J(z) = \left(\int_0^1 \left(U_t^J(z,i) \right)^{\frac{\sigma_{U_t}-1}{\sigma_{U_t}}} di \right)^{\frac{\sigma_{U_t}}{\sigma_{U_t}-1}} \ , \tag{125}$$

where $U_t^J(z,i)$ is the demand by firm z for the labor variety supplied by union i. Given imperfect substitutability between the labor supplied by different unions, they have market power vis-à-vis manufacturing firms. Their demand functions are given by

$$U_t^J(z,i) = \left(\frac{V_t(i)}{V_t}\right)^{-\sigma_{U_t}} U_t^J(z) \; , \tag{126}$$

where $V_t(i)$ is the wage charged to employers by union i and V_t is the aggregate wage paid by employers, given by

$$V_t = \left(\int_0^1 V_t(i)^{1-\sigma_{U_t}} di\right)^{\frac{1}{1-\sigma_{U_t}}} \; . \tag{127}$$

The demand (126) can be aggregated over firms z and sectors J to obtain

$$U_t(i) = \left(\frac{V_t(i)}{V_t}\right)^{-\sigma_{U_t}} U_t \; , \tag{128}$$

where U_t is aggregate labor demand by all manufacturing firms.

GIMF allows for three types of wage rigidities. The first two are the conventional cases of nominal wage rigidities. Sticky wage inflation takes the form familiar from (40):

$$G_{P,t}^U(i) = \frac{\phi_{P^U}}{2} U_t T_t \left(\frac{\frac{V_t(i)}{V_{t-1}(i)}}{\frac{V_{t-1}}{V_{t-2}}} - 1\right)^2 \; . \tag{129}$$

Note that these adjustment costs are zero in steady state even though real wages grow at the rate of world technological progress. Also, the level of world technology enters as a scaling factor in (129), as otherwise these costs would become insignificant over time. The second type of wage rigidities is real wage rigidities, whereby unions resist rapid changes in the real wage V_t/P_t^c. We define $\pi_t^{rw}(i) = \pi_t^v(i)/\left(g\pi_t^C\right)$. Then these adjustment costs are given by

$$G_{P,t}^U(i) = \frac{\phi_{P^U}}{2} U_t T_t \left(\pi_t^{rw}(i) - 1\right)^2 = \frac{\phi_{P^U}}{2} U_t T_t \left(\frac{\frac{V_t(i)}{V_{t-1}(i)}}{g\pi_t^C} - 1\right)^2 \; . \tag{130}$$

The stochastic wage markup of union wages over household wages is given by $\mu_t^U = \sigma_{U_t}/(\sigma_{U_t} - 1)$.

The **optimization** problem of a union consists of maximizing the expected present discounted value of nominal wages paid by firms $V_t(i)U_t(i)$ minus nominal wages paid out to workers $W_tU_t(i)$, minus nominal wage inflation adjustment costs $P_tG_{P,t}^U(i)$. Unlike manufacturers, this sector does not face fixed costs of operation. It is assumed that each union pays out each period's nominal net cash flow as dividends $D_t^U(i)$. The objective function of unions is

$$\underset{\{V_{t+s}(i)\}_{s=0}^{\infty}}{Max} E_t \Sigma_{s=0}^{\infty} \tilde{R}_{t,s} \left[\left(V_{t+s}(i) - W_{t+s} \right) U_{t+s}(i) - V_{t+s} G_{P,t+s}^U(i) \right] \ , \tag{131}$$

subject to labor demands (128) and adjustment costs (129) or (130). We obtain the first order condition for this problem. As all unions face an identical problem, their solutions are identical and the index i can be dropped in all first-order conditions of the problem, with $V_t(i) = V_t$ and $U_t(i) = U_t$. We let $\pi_t^V = V_t/V_{t-1}$, the gross rate of wage inflation, and we rescale by technology. For nominal wage rigidities we obtain the condition

$$\begin{aligned} \left[\mu_t^U \frac{\check{w}_t}{\check{v}_t} - 1 \right] &= \phi_{P^U} \left(\mu_t^U - 1 \right) \left(\frac{\pi_t^V}{\pi_{t-1}^V} \right) \left(\frac{\pi_t^V}{\pi_{t-1}^V} - 1 \right) \\ &- E_t \frac{\theta g n}{\check{r}_{t+1}} \phi_{P^U} \left(\mu_t^U - 1 \right) \frac{\check{v}_{t+1}}{\check{v}_t} \frac{\check{U}_{t+1}}{\check{U}_t} \left(\frac{\pi_{t+1}^V}{\pi_t^V} \right) \left(\frac{\pi_{t+1}^V}{\pi_t^V} - 1 \right) \ . \end{aligned} \tag{132}$$

For real wage rigidities we have

$$\begin{aligned} \left[\mu_t^U \frac{\check{w}_t}{\check{v}_t} - 1 \right] &= \phi_{P^U} \left(\mu_t^U - 1 \right) \pi_t^{rw} \left(\pi_t^{rw} - 1 \right) \\ &- E_t \frac{\theta g n}{\check{r}_{t+1}} \phi_{P^U} \left(\mu_t^U - 1 \right) \frac{\check{v}_{t+1}}{\check{v}_t} \frac{\check{U}_{t+1}}{\check{U}_t} \pi_{t+1}^{rw} \left(\pi_{t+1}^{rw} - 1 \right) \ . \end{aligned} \tag{133}$$

Real **"dividends"** from union organization, denominated in terms of final output, are distributed lump-sum to households in proportion to their share in aggregate labor supply. After rescaling they take the form

$$\check{d}_t^U = (\check{v}_t - \check{w}_t)\check{U}_t - \check{v}_t \check{G}_{P,t}^U \ . \tag{134}$$

We also have $\check{v}_t/\check{v}_{t-1} = (V_t/P_t T_t)/(V_{t-1}/P_{t-1}T_{t-1})$, so that

$$\frac{\check{v}_t}{\check{v}_{t-1}} = \frac{\pi_t^V}{\pi_t g} \ . \tag{135}$$

Finally, the labor **market clearing** condition equates the combined labor supply of OLG and LIQ households to the labor demands coming from nontradables and tradables manufacturers, including their respective labor adjustment costs if applicable, and from unions for wage adjustment costs. We have:

$$\check{L}_t = \check{U}_t^N + \check{U}_t^T + \check{G}_{U,t}^N + \check{G}_{U,t}^T + \check{G}_{P,t}^U \ . \tag{136}$$

10 Import Agents

Each country, in each of its export destination markets, owns two continua of import agents, one for manufactured intermediate tradable goods (T) and another for final goods (D), each indexed by $i \in [0,1]$ and by $J \in \{T, D\}$. Import agents buy intermediate goods (or final goods) from manufacturers (or distributors) in their owners' country and sell these goods to distributors (intermediate goods) or consumption/investment goods producers (final goods) in the destination country. They are perfectly competitive in their input market and monopolistically competitive in their output market. Their price setting is subject to nominal rigidities. We first analyze the demands for their output and then describe their optimization problem.

Demand for the output varieties supplied by import agents comes from distributors (sector T) or consumption/investment goods producers (sectors D), in each case indexed by $z \in [0,1]$. Recall that the domestic economy is indexed by 1 and foreign economies by $j = 2, ..., \tilde{N}$. Domestic distributors z require a separate CES imports aggregate $Y_t^{JM}(1,j,z)$ from the import agents of each country j. That aggregate consists of varieties supplied by different import agents i, $Y_t^{JM}(1,j,z,i)$, with respective prices $P_t^{JM}(1,j,i)$, and is given by

$$Y_t^{JM}(1,j,z) = \left(\int_0^1 \left(Y_t^{JM}(1,j,z,i) \right)^{\frac{\sigma_{JM}-1}{\sigma_{JM}}} di \right)^{\frac{\sigma_{JM}}{\sigma_{JM}-1}} . \tag{137}$$

This gives rise to demands for varieties of

$$Y_t^{JM}(1,j,z,i) = \left(\frac{P_t^{JM}(1,j,i)}{P_t^{JM}(1,j)} \right)^{-\sigma_{JM}} Y_t^{JM}(1,j,z) \; , \tag{138}$$

$$P_t^{JM}(1,j) = \left(\int_0^1 P_t^{JM}(1,j,i)^{1-\sigma_{JM}} di \right)^{\frac{1}{1-\sigma_{JM}}} \; , \tag{139}$$

and these demands can be aggregated over z to yield

$$Y_t^{JM}(1,j,i) = \left(\frac{P_t^{JM}(1,j,i)}{P_t^{JM}(1,j)} \right)^{-\sigma_{JM}} Y_t^{JM}(1,j) \; . \tag{140}$$

Nominal rigidities in this sector take the form familiar from (40),

$$G_{P,t}^{JM}(1,j,i) = \frac{\phi_{P^{JM}}}{2} Y_t^{JM}(1,j) \left(\frac{\frac{P_t^{JM}(1,j,i)}{P_{t-1}^{JM}(1,j,i)}}{\frac{P_{t-1}^{JM}(1,j)}{P_{t-2}^{JM}(1,j)}} - 1 \right)^2 \; , \tag{141}$$

and represent a claim on the underlying exports. Import agents' cost minimizing solution for inputs of manufactured intermediate tradable goods (or final goods) varieties therefore follows equations (34) - (36) above (or similar conditions for demands of consumption/investment goods producers). We denote the price of inputs imported from country j at the border of country 1 by $P_t^{JM,cif}(1,j)$, the cif (cost, insurance, freight) import price. By purchasing power parity this satisfies $P_t^{JM,cif}(1,j) = \tilde{p}_t^{exp} P_t^{JH}(j)\mathcal{E}_t(1)/\mathcal{E}_t(j)$, where $\tilde{p}_t^{exp}$ is an exogenous price shock that equals the inverse of a shock to the technology that converts foreign exports into domestic imports. In real terms we have

$$p_t^{JM,cif}(1,j) = p_t^{JH}(j)\tilde{p}_t^{exp}(j)\frac{e_t(1)}{e_t(j)} \ . \tag{142}$$

The **optimization** problem of import agents consists of maximizing the expected present discounted value of nominal revenue $P_t^{JM}(1,j,i)Y_t^{JM}(1,j,i)$ minus nominal costs of inputs $P_t^{JM,cif}(1,j)Y_t^{JM}(1,j,i)$, minus nominal inflation adjustment costs $P_t G_{P,t}^{JM}(1,j,i)$. The latter represent a demand for final output. This sector does not face fixed costs of operation. It is assumed that each import agent pays out each period's nominal net cash flow as dividends $D_t^{JM}(1,j,i)$. The objective function of import agents is

$$\underset{\left\{P_{t+s}^{JM}(1,j,i)\right\}_{s=0}^{\infty}}{Max} E_t \Sigma_{s=0}^{\infty} \tilde{R}_{t,s} \left[\left(P_{t+s}^{JM}(1,j,i) - P_{t+s}^{JM,cif}(1,j) \right) Y_{t+s}^{JM}(1,j,i) - P_{t+s}^{JM} G_{P,t+s}^{JM}(1,j,i) \right] \ , \tag{143}$$

subject to demands (140) and adjustment costs (141). The first order condition for this problem, after dropping firm specific subscripts and rescaling by technology, has the form:

$$\left[\frac{\sigma_{JM}}{\sigma_{JM}-1} \frac{p_t^{JM,cif}(1,j)}{p_t^{JM}(1,j)} - 1 \right] = \frac{\phi_{P^{JM}}}{\sigma_{JM}-1} \left(\frac{\pi_t^{JM}(1,j)}{\pi_{t-1}^{JM}(1,j)} \right) \left(\frac{\pi_t^{JM}(1,j)}{\pi_{t-1}^{JM}(1,j)} - 1 \right) \tag{144}$$
$$-E_t \frac{\theta g n}{\check{r}_{t+1}} \frac{\phi_{P^{JM}}}{\sigma_{JM}-1} \frac{p_{t+1}^{JM}(1,j)}{p_t^{JM}(1,j)} \frac{\check{Y}_{t+1}^{JM}(1,j)}{\check{Y}_t^{JM}(1,j)} \left(\frac{\pi_{t+1}^{JM}(1,j)}{\pi_t^{JM}(1,j)} \right) \left(\frac{\pi_{t+1}^{JM}(1,j)}{\pi_t^{JM}(1,j)} - 1 \right) \ .$$

The rescaled real dividends of country j's import agent in the domestic economy, which are paid out to OLG households in country j, are

$$\check{d}_t^{JM}(1,j) = (p_t^{JM}(1,j) - p_t^{JM,cif}(1,j))\check{Y}_t^{JM}(1,j) - p_t^{JM}(1,j)\check{G}_{P,t}^{JM}(1,j) \ . \tag{145}$$

The total dividends received by OLG households in country 1, expressed in terms of country 1 output, are

$$\check{d}_t^{JM} = \check{d}_t^{JM}(1) = \Sigma_{j=2}^{\tilde{N}} \check{d}_t^{JM}(j,1) \frac{e_t(1)}{e_t(j)} \ , \tag{146}$$

$$\check{d}_t^{M} = \check{d}_t^{TM} + \check{d}_t^{DM} \ . \tag{147}$$

Finally, the **market clearing** conditions for import agents equate the export volume received from abroad to the import volume used domestically plus adjustment costs:

$$\check{Y}_t^{JX}(j,1) = \check{Y}_t^{JM}(1,j) + \check{G}_{P,t}^{JM}(1,j) \ . \tag{148}$$

11 Distributors

Distributors produce domestic final output. They buy domestic tradables and nontradables from domestic manufacturers, and foreign tradables from import agents. They also use the stock of public infrastructure free of a user charge. Distributors sell their final output composite to consumption goods producers, investment goods producers and final goods import agents in foreign countries. They are perfectly competitive in both their output and input markets.

We divide our description of the **technology** of distributors into a number of stages. In the first stage a foreign input composite is produced from intermediate manufactured inputs originating in all foreign economies and sold to distributors by import agents. In the second stage a tradables composite is produced by combining these foreign tradables with domestic tradables, subject to an adjustment cost that makes rapid changes in the share of foreign tradables costly. In the third stage a tradables-nontradables composite is produced. In the fourth stage the tradables-nontradables composite is combined with a publicly provided stock of infrastructure.

Foreign input composites $Y_t^{JF}(1)$, $J \in \{T, D\}$, are produced by combining imports $Y_t^{JM}(1,j)$ originating in different foreign economies j and purchased through import agents. A foreign input choice problem therefore only arises when there are more than 2 countries. Also, distributors use only the composite indexed by T, while the composite indexed by D is used by consumption and investment goods manufacturers. We present the problem here in its general form and then reapply the results when describing these other agents. The CES production function for $Y_t^{JF}(1)$ has an elasticity of substitution ξ_{JM} and share parameters $\zeta^J(1,j)$ that are identical across firms and that add up to one, $\Sigma_{j=2}^{\tilde{N}}\zeta^J(1,j) = 1$. We also allow for an additional effect of technology shocks on the intermediates import share parameters. Specifically, we posit that an improvement in technology in a foreign country not only leads to a lower cost in that country, but also to a higher demand for the respective good in all foreign countries, reflecting quality improvements due to better technology. The import share parameter between countries 1 and j is therefore given by

$$\tilde{\zeta}^T(1,j) = \left(\frac{\zeta^T(1,j) A_t^T(j)^{\varkappa(1)}}{\tilde{\zeta}^T(1)} \right) , \tag{149}$$

$$\tilde{\zeta}^T(1) = \Sigma_{j=2}^{\tilde{N}}\zeta^T(1,j) A_t^T(j)^{\varkappa(1)} , \tag{150}$$

where $\varkappa = 0$ corresponds to the standard case while $\varkappa > 0$ introduces positive foreign demand effects of technological progress. This makes it more likely that technological progress in the tradables sector will lead to a real appreciation. By contrast, for investment and consumption goods producers we assume $\tilde{\zeta}^D(1,j) = \zeta^D(1,j)$. The local currency prices $P_t^{JM}(1,j)$ of imports in country 1 are determined by import agents, and the overall cost of the bundle $Y_t^{JF}(1)$ is $P_t^{JF}(1)$. In the calibration of the model the share parameters $\zeta^J(1,j)$ will be parameterized using a multi-region trade matrix. We have the following sub-production function:

$$Y_t^{JF}(1) = \left(\Sigma_{j=2}^{N} \tilde{\zeta}^J(1,j)^{\frac{1}{\xi_{JM}}} \left(Y_t^{JM}(1,j) \right)^{\frac{\xi_{JM}-1}{\xi_{JM}}} \right)^{\frac{\xi_{JM}}{\xi_{JM}-1}} , \tag{151}$$

with demands

$$Y_t^{JM}(1,j) = \tilde{\zeta}^J(1,j) Y_t^{JF}(1) \left(\frac{P_t^{JM}(1,j)}{P_t^{JF}(1)} \right)^{-\xi_{JM}} \tag{152}$$

and an import price index, written in terms of relative prices, of

$$p_t^{JF}(1) = \left(\Sigma_{j=2}^{N} \tilde{\zeta}^J(1,j) \left(p_t^{JM}(1,j) \right)^{1-\xi_{JM}} \right)^{\frac{1}{1-\xi_{JM}}} . \tag{153}$$

Equations (151) and (152) are rescaled by technology and population to generate aggregate foreign input demand of country 1, $\check{Y}_t^{JF}(1)$ and aggregate demands for individual country imports $\check{Y}_t^{JM}(1,j)$. Note that for final goods $\check{Y}_t^{DF}$ there is a market clearing condition because the imported bundle is sold to both consumption and investment goods producers:

$$\check{Y}_t^{DF} = \check{Y}_t^{CF} + \check{Y}_t^{IF} . \tag{154}$$

In the two country case equations (151)-(153) simplify, after aggregation, to $\check{Y}_t^{JF}(1) = \check{Y}_t^{JM}(1,2)$ and $p_t^{JF} = p_t^{JM}$. In our notation we will now revert to the two-country case and drop the index 1 for Home.

The **tradables composite** Y_t^T is produced by combining foreign produced tradables Y_t^{TF} with domestically produced tradables Y_t^{TH}, in a CES technology with elasticity of substitution ξ_T. A key concern in open economy DSGE models is the potential for an excessive short-term responsiveness of international trade to real exchange rate movements. This model avoids that problem by introducing adjustment costs $G_{F,t}^T$ that make it costly to vary the share of Foreign produced tradables in total tradables production Y_t^{TF}/Y_t^T relative to the value of that share in the aggregate distribution sector in the previous period Y_{t-1}^{TF}/Y_{t-1}^T. At the previous level we allowed for the possibility $\varkappa > 0$, meaning foreign technology shocks affect relative demands for goods from different countries. We allow for an identical effect, dependent on the same parameter, to affect relative demands for domestic and foreign tradable goods. Specifically, an improvement in average world technology increases the relative demand for foreign produced tradables. The domestic and foreign tradables share parameters are therefore given by

$$\tilde{\alpha}_{H_t}^T = \frac{\alpha_{H_t}^T \left(A_t^T\right)^{\varkappa}}{\breve{\alpha}_{H_t}^T} , \tag{155}$$

$$\tilde{\alpha}_{TF_t} = \frac{\left(1-\alpha_{H_t}^T\right)\left(A_t^{RW}\right)^{\varkappa}}{\breve{\alpha}_{H_t}^T} , \tag{156}$$

$$\breve{\alpha}_H^T = \alpha_{H_t}^T \left(A_t^T\right)^{\varkappa} + \left(1-\alpha_{H_t}^T\right)\left(A_t^{RW}\right)^{\varkappa} , \tag{157}$$

$$A_t^{RW} = \Sigma_{j=2}^{\tilde{N}} A_t^T(j)^{\frac{gdp_{ss}(j)}{\Sigma_{k=2}^{\tilde{N}} gdp_{ss}(k)}} . \tag{158}$$

The sub-production function for tradables then has the following form:[23,24]

$$Y_t^T = \left(\left(\tilde{\alpha}_{H_t}^T\right)^{\frac{1}{\xi_T}} \left(Y_t^{TH}\right)^{\frac{\xi_T-1}{\xi_T}} + \left(\tilde{\alpha}_{F_t}^T\right)^{\frac{1}{\xi_T}} \left(Y_t^{TF}(1-G_{F,t}^T)\right)^{\frac{\xi_T-1}{\xi_T}} \right)^{\frac{\xi_T}{\xi_T-1}} , \tag{159}$$

$$G_{F,t}^T = \frac{\phi_{FT}}{2} \frac{\left(\mathcal{R}_t^T - 1\right)^2}{1+\left(\mathcal{R}_t^T - 1\right)^2} , \tag{160}$$

23 Home bias in tradables use depends on the parameter α_{TH} and on a similar parameter α_{DH} at the level of final goods imports.

24 For the ratio $\mathcal{R}_t^T$ we assume as usual that the distributor takes the lagged denominator term as given in his optimization.

$$\mathcal{R}_t^T = \frac{\frac{Y_t^{TF}}{Y_t^T}}{\frac{Y_{t-1}^{TF}}{Y_{t-1}^T}} \ . \tag{161}$$

After expressing prices in terms of the numeraire, and after rescaling by technology and population, we obtain the aggregate tradables sub-production function from (159) - (161). We also obtain the following first-order conditions for optimal input choice:

$$\check{Y}_t^{TH} = \tilde{\alpha}_{H_t}^T \check{Y}_t^T \left(\frac{p_t^{TH}}{p_t^T} \right)^{-\xi_T} \ , \tag{162}$$

$$\check{Y}_t^{TF} \left[1 - G_{F,t}^T \right] = \tilde{\alpha}_{F_t}^T \check{Y}_t^T \left(\frac{p_t^{TF}}{p_t^T} \right)^{-\xi_T} \left(\tilde{O}_t^T \right)^{\xi_T} \ , \tag{163}$$

$$\tilde{O}_t^T = 1 - G_{F,t}^T - \phi_{FT} \frac{\mathcal{R}_t^T \left(\mathcal{R}_t^T - 1 \right)}{\left[1 + \left(\mathcal{R}_t^T - 1 \right)^2 \right]^2} \ . \tag{164}$$

The **tradables-nontradables composite** Y_t^A is produced with another CES production function with elasticity of substitution ξ_A. We again allow for a relative demand effect, this time of nontradables productivity shocks, with input share parameters given by

$$\tilde{\alpha}_{T_t} = \frac{(1 - \alpha_N)}{\breve{\alpha}_{N_t}} \ , \tag{165}$$

$$\tilde{\alpha}_{N_t} = \frac{\alpha_N \left(A_t^N \right)^{\tilde{\varkappa}}}{\breve{\alpha}_{N_t}} \ , \tag{166}$$

$$\breve{\alpha}_{N_t} = \alpha_N \left(A_t^N \right)^{\tilde{\varkappa}} + (1 - \alpha_N) \ . \tag{167}$$

The sub-production function for the tradables-nontradables composite then has the following form:

$$Y_t^A = \left((\tilde{\alpha}_{T_t})^{\frac{1}{\xi_A}} \left(Y_t^T \right)^{\frac{\xi_A - 1}{\xi_A}} + (\tilde{\alpha}_{N_t})^{\frac{1}{\xi_A}} \left(Y_t^N \right)^{\frac{\xi_A - 1}{\xi_A}} \right)^{\frac{\xi_A}{\xi_A - 1}} \ . \tag{168}$$

The real marginal cost of producing Y_t^A is, with obvious notation for sectorial price levels,

$$p_t^A = \left[\tilde{\alpha}_{T_t} \left(p_t^T \right)^{1 - \xi_A} + \tilde{\alpha}_{N_t} \left(p_t^N \right)^{1 - \xi_A} \right]^{\frac{1}{1 - \xi_A}} \ . \tag{169}$$

After expressing prices in terms of the numeraire, and after rescaling by technology, we obtain the aggregate tradables-nontradables sub-production function from (168), and the following first-order conditions for optimal input choice:

$$\check{Y}_t^N = \tilde{\alpha}_{N_t} \check{Y}_t^A \left(\frac{p_t^N}{p_t^A} \right)^{-\xi_A} , \tag{170}$$

$$\check{Y}_t^T = \tilde{\alpha}_{T_t} \check{Y}_t^A \left(\frac{p_t^T}{p_t^A} \right)^{-\xi_A} . \tag{171}$$

For the case where the nontradables sector is excluded from GIMF, we simply have $\check{Y}_t^A = \check{Y}_t^T$ and $p_t^A = p_t^T$.

The **private-public composite** Z_t^D, which we will refer to as domestic final output, is produced with the following production function:

$$Z_t^D = Y_t^A \left(K_t^{G1} \right)^{\alpha_{G1}} \left(K_t^{G2} \right)^{\alpha_{G2}} \mathcal{S} . \tag{172}$$

The inputs are the tradables-nontradables composite Y_t^A and the stocks of public capital K_t^{G1} and K_t^{G2}, which are identical for all firms and provided free of charge to the end user (but not of course to the taxpayer). Note that this production function exhibits constant returns to scale in private inputs while the public capital stocks enter externally, in an analogous manner to exogenous technology. The term $\mathcal{S}$ is a technology scale factor that can be used to normalize steady state technology to one, $\left(\bar{K}^{G1} \right)^{\alpha_{G1}} \left(\bar{K}^{G2} \right)^{\alpha_{G2}} \mathcal{S} = 1$.

The real marginal cost of Z_t^D is denoted as p_t^{DH}, while the real marginal cost of Y_t^A is p_t^A. After expressing prices in terms of the numeraire, and after rescaling by technology and population, we obtain the normalized production function from (172), and the following first-order condition:

$$p_t^{DH} \left(\check{K}_t^{G1} \right)^{\alpha_{G1}} \left(\check{K}_t^{G2} \right)^{\alpha_{G2}} \mathcal{S} = p_t^A . \tag{173}$$

The rescaled aggregate **dividends** of distributors (equal to zero in equilibrium) are

$$\check{d}_t^D = p_t^{DH} \check{Z}_t^D - p_t^N \check{Y}_t^N - p_t^{TH} \check{Y}_t^{TH} - p_t^{TF} \check{Y}_t^{TF} . \tag{174}$$

Finally, the **market clearing** conditions for this sector equates its output to the demands of consumption and investment goods producers and of foreign import agents:

$$\check{Z}_t^D = \check{Y}_t^{IH} + \check{Y}_t^{CH} + \tilde{p}_t^{exp} \Sigma_{j=2}^{\tilde{N}} \check{Y}_t^{DX}(1,j) . \tag{175}$$

12 Investment Goods Producers

Investment goods producers buy domestic final output directly from domestic distributors, and foreign final output indirectly via import agents. They sell the final composite Z_t^I to manufacturers (in their role as investors), to the government, and back to other investment goods producers for the purpose of fixed and adjustment costs. There is a continuum of investment goods producers indexed by $i \in [0,1]$. They are perfectly competitive in their input markets and monopolistically competitive in their output market. Their price setting is subject to nominal rigidities. We first analyze the demand for their output, then we turn to their technology, and finally we describe their profit maximization problem.

Demand for investment goods varieties comes from multiple sources. Let z be an individual purchaser of investment goods. Then his demand $\mathcal{D}_t^I(z)$ is for a CES composite of investment goods varieties i, with time-varying elasticity of substitution σ_{I_t}

$$\mathcal{D}_t^I(z) = \left(\int_0^1 \left(\mathcal{D}_t^I(z,i) \right)^{\frac{\sigma_{I_t}-1}{\sigma_{I_t}}} di \right)^{\frac{\sigma_{I_t}}{\sigma_{I_t}-1}} , \tag{176}$$

with associated demands

$$\mathcal{D}_t^I(z,i) = \left(\frac{P_t^I(i)}{P_t^I} \right)^{-\sigma_{I_t}} \mathcal{D}_t^I(z) , \tag{177}$$

where $P_t^I(i)$ is the price of variety i of investment goods output, and P_t^I is the aggregate investment goods price level given by

$$P_t^I = \left(\int_0^1 \left(P_t^I(i) \right)^{1-\sigma_{I_t}} di \right)^{\frac{1}{1-\sigma_{I_t}}} . \tag{178}$$

Furthermore, the total demand facing a producer of investment goods variety i can be obtained by aggregating over all sources of demand z. We obtain

$$\mathcal{D}_t^I(i) = \left(\frac{P_t^I(i)}{P_t^I} \right)^{-\sigma_{I_t}} \mathcal{D}_t^I , \tag{179}$$

where $\mathcal{D}_t^I(i)$ and $\mathcal{D}_t^I$ remain to be specified by way of a market clearing condition for investment goods output. The exogenous and stochastic price markup is given by $\mu_t^I = \sigma_{I_t}/(\sigma_{I_t}-1)$.

The **technology** of investment goods producers consists of a CES production function that uses domestic final output $Y_t^{IH}(i)$ and foreign final output imported via import agents $Y_t^{IF}(i)$, with a share coefficient for domestic final output of $\alpha_{H_t}^I$ and an elasticity of substitution ξ_I. There is an adjustment cost $G^I_{F,t}$ that makes it costly to vary the share of foreign inputs $Y^{IF}_t(i)/Z^I_t(i)$ relative to the value of that share in the aggregate investment goods distribution sector in the previous period Y^{IF}_{t-1}/Z^I_{t-1}. We therefore have

$$Z^I_t(i) = \left(\left(\alpha^I_{H_t}\right)^{\frac{1}{\xi_I}} \left(Y^{IH}_t(i)\right)^{\frac{\xi_I - 1}{\xi_I}} + \left(1-\alpha^I_{H_t}\right)^{\frac{1}{\xi_I}} \left(Y^{IF}_t(i)(1-G^I_{F,t}(i))\right)^{\frac{\xi_I - 1}{\xi_I}} \right)^{\frac{\xi_I}{\xi_I - 1}} , \tag{180}$$

$$G^I_{F,t}(i) = \frac{\phi_{FI}}{2} \frac{\left(\mathcal{R}^I_t - 1\right)^2}{1 + \left(\mathcal{R}^I_t - 1\right)^2} , \tag{181}$$

$$\mathcal{R}^I_t = \frac{\frac{Y^{IF}_t(i)}{Z^I_t(i)}}{\frac{Y^{IF}_{t-1}}{Z^I_{t-1}}} . \tag{182}$$

After expressing prices in terms of the numeraire, and after rescaling by technology and population, we obtain the aggregate investment goods production function from (180) - (182). Letting the marginal cost of producing Z^I_t be denoted by p^{II}_t, we also obtain the following first-order conditions for optimal input choice:

$$\check{Y}^{IH}_t = \alpha^I_{H_t} \check{Z}^I_t \left(\frac{p^{DH}_t}{p^{II}_t}\right)^{-\xi_I} , \tag{183}$$

$$\check{Y}^{IF}_t \left[1 - G^I_{F,t}\right] = \left(1-\alpha^I_{H_t}\right) \check{Z}^I_t \left(\frac{p^{DF}_t}{p^{II}_t}\right)^{-\xi_I} \left(\tilde{O}^I_t\right)^{\xi_I} , \tag{184}$$

$$\tilde{O}^I_t = 1 - G^I_{F,t} - \phi_{FI} \frac{\mathcal{R}^I_t \left(\mathcal{R}^I_t - 1\right)}{\left[1 + \left(\mathcal{R}^I_t - 1\right)^2\right]^2} . \tag{185}$$

We finally turn to the **profit maximization** problem. It consists of maximizing the expected present discounted value of nominal revenue $P^{ZI}_t(i)\mathcal{D}^I_t(i)$ minus nominal costs of production $P^{II}_t\mathcal{D}^I_t(i)$, a fixed cost $P^{ZI}_t T_t \omega^I$, and inflation adjustment costs $P^{ZI}_t G^I_{P,t}(i)$. The latter are real resource costs that have to be paid out of investment goods output Z^I_t. Their functional form is by now familiar:

$$G^I_{P,t}(i) = \frac{\phi_{P^I}}{2} \mathcal{D}^I_t \left(\frac{\frac{P^{ZI}_t(i)}{P^{ZI}_{t-1}(i)}}{\frac{P^{ZI}_{t-1}}{P^{ZI}_{t-2}}} - 1 \right)^2 . \tag{186}$$

Fixed costs are given by

$$\omega^I = \bar{Z}^I \frac{\bar{\mu}^I - 1}{\bar{\mu}^I} \left(1 - s_\pi\right) . \tag{187}$$

It is assumed that the producer pays out each period's nominal net cash flow as dividends $D_t^I(i)$. The objective function is

$$\underset{\left\{P_{t+s}^{ZI}(i)\right\}_{s=0}^{\infty}}{Max} E_t \Sigma_{s=0}^{\infty} \tilde{R}_{t,s} \left[\left(P_{t+s}^{ZI}(i) - P_{t+s}^{II} \right) \mathcal{D}_{t+s}^I(i) - P_{t+s}^{ZI} G_{P,t+s}^I(i) - P_{t+s}^{ZI} T_{t+s} \omega^I \right] \ , \tag{188}$$

subject to product demands (179) and given marginal cost P_t^{II}. We obtain the first order condition for this problem, again using the fact that all firms behave identically in equilibrium. Using the equilibrium condition $\mathcal{D}_t^I = Z_t^I$ we obtain

$$\begin{aligned} \left[\mu_t^I \frac{p_t^{II}}{p_t^{ZI}} - 1 \right] &= \phi_{P^I} \left(\mu_t^I - 1 \right) \left(\frac{\pi_t^{ZI}}{\pi_{t-1}^{ZI}} \right) \left(\frac{\pi_t^{ZI}}{\pi_{t-1}^{ZI}} - 1 \right) \\ &- E_t \frac{\theta g n}{\check{r}_{t+1}} \phi_{P^I} \left(\mu_t^I - 1 \right) \frac{p_{t+1}^{ZI}}{p_t^{ZI}} \frac{\check{Z}_{t+1}^I}{\check{Z}_t^I} \left(\frac{\pi_{t+1}^{ZI}}{\pi_t^{ZI}} \right) \left(\frac{\pi_{t+1}^{ZI}}{\pi_t^{ZI}} - 1 \right) \ . \end{aligned} \tag{189}$$

The rescaled aggregate **dividends** of investment goods producers are

$$\check{d}_t^I = p_t^{ZI} \left(\check{Z}_t^I - \check{G}_{P,t}^I - \omega^I \right) - p_t^{DH} \check{Y}_t^{IH} - p_t^{DF} \check{Y}_t^{IF} \ . \tag{190}$$

Finally, we allow for unit root and stationary **shocks to the relative price of investment goods**. Specifically, the net output of investment goods producers,

$$\check{X}_t^I = \check{Z}_t^I - \check{G}_{P,t}^I - \omega^I \ , \tag{191}$$

is converted to final output of investment goods $\check{Y}_t^I$ using the technology

$$\check{Y}_t^I = A_t^I T_t^I \check{X}_t^I \ , \tag{192}$$

where A_t^I is a stationary technology shock and T_t^I is a unit root technology shock with zero trend growth. We define the relative price terms $\tilde{p}_t^I = 1/T_t^I$ and $\breve{p}_t^I = 1/A_t^I$. Competitive pricing means that the price of final investment goods equals

$$p_t^I = \tilde{p}_t^I \breve{p}_t^I p_t^{ZI} \ . \tag{193}$$

The **market clearing** condition for investment goods therefore equates output to the demands of manufacturers (as investors) or capital producers, the government, and the investment goods producers themselves for fixed and adjustment costs:

$$\check{Z}_t^I - \check{G}_{P,t}^I - \omega^I = \tilde{p}_t^I \breve{p}_t^I \left(\check{I}_t + \check{G}_{I,t}^N + + \check{G}_{I,t}^T + \check{Y}_t^{GI} \right) \ . \tag{194}$$

13 Consumption Goods Producers

Consumption goods producers buy domestic final output directly from domestic distributors, and foreign final output indirectly via import agents. They sell the final composite Z_t^C to consumption goods retailers, to the government, and back to other consumption goods producers for the purpose of fixed and adjustment costs. There is a continuum of consumption goods producers indexed by $i \in [0,1]$. They are perfectly competitive in their input markets and monopolistically competitive in their output market. Their price setting is subject to nominal rigidities. We first analyze the demand for consumption goods, then we turn to consumption goods producers' technology, and finally we describe their profit maximization problem.

Demand for the consumption goods varieties comes from multiple sources. Let z be an individual purchaser of consumption goods. Then his demand $\mathcal{D}_t^C(z)$ is for a CES composite of final output varieties i, with time-varying elasticity of substitution σ_{C_t}:

$$\mathcal{D}_t^C(z) = \left(\int_0^1 \left(\mathcal{D}_t^C(z,i) \right)^{\frac{\sigma_{C_t}-1}{\sigma_{C_t}}} di \right)^{\frac{\sigma_{C_t}}{\sigma_{C_t}-1}} , \tag{195}$$

with associated demands

$$\mathcal{D}_t^C(z,i) = \left(\frac{P_t(i)}{P_t} \right)^{-\sigma_{C_t}} \mathcal{D}_t^C(z) , \tag{196}$$

where $P_t(i)$ is the price of variety i of consumption goods output, and P_t is the aggregate consumption goods price level given by

$$P_t = \left(\int_0^1 \left(P_t(i) \right)^{1-\sigma_{C_t}} di \right)^{\frac{1}{1-\sigma_{C_t}}} . \tag{197}$$

We choose this price level as the economy's numeraire. The total demand facing a producer of consumption goods variety i can be obtained by aggregating over all sources of demand z. We obtain

$$\mathcal{D}_t^C(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\sigma_{C_t}} \mathcal{D}_t^C , \tag{198}$$

where $\mathcal{D}_t^C(i)$ and $\mathcal{D}_t^C$ remain to be specified by way of a market clearing condition for consumption goods output. The exogenous and stochastic price markup is given by $\mu_t^C = \sigma_{C_t}/(\sigma_{C_t}-1)$.

The **technology** of consumption goods producers consists of a CES production function that uses domestic final output $Y_t^{CH}(i)$ and foreign final output imported via import agents $Y_t^{CF}(i)$, with a share coefficient for domestic final output of $\alpha^C_{H_t}$ and an elasticity of substitution ξ_C. As for foreign final output imports, there is an adjustment cost $G^C_{F,t}$ that makes it costly to vary the share of foreign inputs $Y^{CF}_t(i)/Z^C_t(i)$ relative to the value of that share in the aggregate consumption goods distribution sector in the previous period Y^{CF}_{t-1}/Z^C_{t-1}. We therefore have

$$Z^C_t(i) = \left(\left(\alpha^C_{H_t}\right)^{\frac{1}{\xi_C}} \left(Y^{CH}_t(i)\right)^{\frac{\xi_C - 1}{\xi_C}} + \left(1 - \alpha^C_{H_t}\right)^{\frac{1}{\xi_C}} \left(Y^{CF}_t(i)(1 - G^C_{F,t}(i))\right)^{\frac{\xi_C - 1}{\xi_C}} \right)^{\frac{\xi_C}{\xi_C - 1}} , \quad (199)$$

$$G^C_{F,t}(i) = \frac{\phi_{FC}}{2} \frac{\left(\mathcal{R}^C_t - 1\right)^2}{1 + \left(\mathcal{R}^C_t - 1\right)^2} , \quad (200)$$

$$\mathcal{R}^C_t = \frac{\frac{Y^{CF}_t(i)}{Z^C_t(i)}}{\frac{Y^{CF}_{t-1}}{Z^C_{t-1}}} . \quad (201)$$

After expressing prices in terms of the numeraire, and after rescaling by technology and population, we obtain the aggregate consumption goods production function from (199) - (201). Letting the marginal cost of producing Z^C_t be denoted by p^{CC}_t, we also obtain the following first-order conditions for optimal input choice:

$$\check{Y}^{CH}_t = \alpha^C_{H_t} \check{Z}^C_t \left(\frac{p^{DH}_t}{p^{CC}_t}\right)^{-\xi_C} , \quad (202)$$

$$\check{Y}^{CF}_t \left[1 - G^C_{F,t}\right] = \left(1 - \alpha^C_{H_t}\right) \check{Z}^C_t \left(\frac{p^{DF}_t}{p^{CC}_t}\right)^{-\xi_C} \left(\tilde{O}^C_t\right)^{\xi_C} , \quad (203)$$

$$\tilde{O}^C_t = 1 - G^C_{F,t} - \phi_{FC} \frac{\mathcal{R}^C_t \left(\mathcal{R}^C_t - 1\right)}{\left[1 + \left(\mathcal{R}^C_t - 1\right)^2\right]^2} . \quad (204)$$

We finally turn to the **profit maximization** problem. It consists of maximizing the expected present discounted value of nominal revenue $P_t(i)\mathcal{D}^C_t(i)$ minus nominal costs of production $P^{CC}_t \mathcal{D}^C_t(i)$, a fixed cost $P_t T_t \omega^C$, and inflation adjustment costs $P_t G^C_{P,t}(i)$. The latter are real resource costs that have to be paid out of consumption goods output Z^C_t. Their functional form is the familiar

$$G^C_{P,t}(i) = \frac{\phi_{PC}}{2} \mathcal{D}^C_t \left(\frac{\frac{P_t(i)}{P_{t-1}(i)}}{\frac{P_{t-1}}{P_{t-2}}} - 1 \right)^2 . \quad (205)$$

Fixed costs are given by

$$\omega^C = \bar{Z}^C \frac{\bar{\mu}^C - 1}{\bar{\mu}^C} \left(1 - s_\pi\right) . \quad (206)$$

It is assumed that the producer pays out each period's nominal net cash flow as dividends $D_t^C(i)$. The objective function is

$$\underset{\{P_{t+s}(i)\}_{s=0}^{\infty}}{Max} E_t \Sigma_{s=0}^{\infty} \tilde{R}_{t,s} \left[\left(P_{t+s}(i) - P_{t+s}^{CC} \right) \mathcal{D}_{t+s}^C(i) - P_{t+s} G_{P,t+s}^C(i) - P_{t+s} T_{t+s} \omega^C \right] \ , \tag{207}$$

subject to product demands (198) and given marginal cost P_t^{CC}. We obtain the first order condition for this problem, again using the fact that all firms behave identically in equilibrium. Using the equilibrium condition $\mathcal{D}_t^C = Z_t^C$ we obtain

$$\begin{aligned} \left[\mu_t^C p_t^{CC} - 1 \right] &= \phi_{P^C} \left(\mu_t^C - 1 \right) \left(\frac{\pi_t}{\pi_{t-1}} \right) \left(\frac{\pi_t}{\pi_{t-1}} - 1 \right) \\ &- E_t \frac{\theta g n}{\check{r}_{t+1}} \phi_{P^C} \left(\mu_t^C - 1 \right) \frac{\check{Z}_{t+1}^C}{\check{Z}_t^C} \left(\frac{\pi_{t+1}}{\pi_t} \right) \left(\frac{\pi_{t+1}}{\pi_t} - 1 \right) \ . \end{aligned} \tag{208}$$

The rescaled aggregate **dividends** of consumption goods producers are

$$\check{d}_t^C = \check{Z}_t^C - p_t^{DH} \check{Y}_t^{CH} - p_t^{DF} \check{Y}_t^{CF} - \check{G}_{P,t}^C - \omega^C \ . \tag{209}$$

The **market clearing** condition for consumption goods equates output to the demands of consumption goods retailers, the government, and the consumption goods producers themselves for fixed and adjustment costs:

$$\check{Z}_t^C = \check{C}_t^{ret} + \check{Y}_t^{GC} + \omega^C + \check{G}_{P,t}^C + \check{G}_{C,t} \ . \tag{210}$$

14 Retailers

There is a continuum of retailers indexed by $i \in [0, 1]$. Retailers combine final output purchased from consumption goods producers and raw materials purchased from raw materials producers, where there are adjustment costs to rapid changes in raw materials inputs. Retailers sell their output to households. They are perfectly competitive in their input market and monopolistically competitive in their output market. Their price setting is subject to real rigidities in that they find it costly to rapidly adjust their sales volume to changing demand conditions. We first analyze retailers' technology, then the demands for their output, and finally their optimization problem.

The **technology** of each retailer is given by a CES production function in consumption goods $C_t^{ret}(i)$ and directly consumed raw materials $X_t^C(i)$, with elasticity of substitution ξ_{XC}. An adjustment cost $G^{C}_{X,t}(i)$ makes fast changes in raw materials inputs costly. We have

$$C_t(i) = \left(\left(1-\alpha^X_{C_t}\right)^{\frac{1}{\xi_{XC}}} \left(C^{ret}_t(i)\right)^{\frac{\xi_{XC}-1}{\xi_{XC}}} + \left(\alpha^X_{C_t}\right)^{\frac{1}{\xi_{XC}}} \left(X^C_t(i)\left(1-G^C_{X,t}(i)\right)\right)^{\frac{\xi_{XC}-1}{\xi_{XC}}} \right)^{\frac{\xi_{XC}}{\xi_{XC}-1}} , \tag{211}$$

$$G^C_{X,t}(i) = \frac{\phi^C_X}{2} \left(\frac{\left(X^C_t(i)/\left(gn\right)\right) - X^C_{t-1}}{X^C_{t-1}} \right)^2 . \tag{212}$$

The optimal input choice for this problem, after normalizing by technology and population, and after dropping the agent specific index i, is given by

$$\frac{\check{X}^C_t}{\check{C}^{ret}_t} = \frac{\alpha^X_{C_t}}{\left(1-\alpha^X_{C_t}\right)\left(1-G^C_{X,t}\right)} \left(\frac{p^X_t}{\tilde{O}^C_t}\right)^{-\xi_{XC}} ,$$

$$\tilde{O}^C_t = \left(1 - G^C_{X,t} - \phi^C_X \frac{\check{X}^C_t}{\check{X}^C_{t-1}} \left(\frac{\check{X}^C_t - \check{X}^C_{t-1}}{\check{X}^C_{t-1}} \right)\right) , \tag{213}$$

and marginal cost is

$$p^C_t = \left(\left(1-\alpha^X_{C_t}\right) + \alpha^X_{C_t} \left(\frac{p^X_t}{\tilde{O}^C_t}\right)^{1-\xi_{XC}} \right)^{\frac{1}{1-\xi_{XC}}} . \tag{214}$$

When the raw materials sector is excluded from GIMF, the above simplifies to $\check{C}_t = \check{C}^{ret}_t$ and $p^C_t = 1$.

Demand for the output varieties $C_t(i)$ supplied by retailers comes from households, and follows directly from (10) and (29) as

$$C_t(i) = \left(\frac{P^R_t(i)}{P^R_t}\right)^{-\sigma_R} C_t . \tag{215}$$

The **optimization** problem of retailers consists of maximizing the expected present discounted value of nominal revenue $P^R_t(i)C_t(i)$ minus nominal costs of inputs $P^C_t C_t(i)$, minus nominal quantity adjustment costs $P_t G_{C,t}(i)$, where the latter represent a demand for consumption goods output. This sector does not face fixed costs of operation. The quantity adjustment costs take the form[25]

$$G_{C,t}(i) = \frac{\phi_C}{2} C_t \left(\frac{\left(C_t(i)/(gn)\right) - C_{t-1}(i)}{C_{t-1}(i)} \right)^2 . \tag{216}$$

It is assumed that each retailer pays out each period's nominal net cash flow as dividends $D^R_t(i)$. The objective function of retailers is

$$\underset{\left\{P^R_{t+s}(i)\right\}_{s=0}^{\infty}}{Max} \; E_t \Sigma_{s=0}^{\infty} \tilde{R}_{t,s} \left[\left(P^R_{t+s}(i) - P^C_{t+s}\right) C_{t+s}(i) - P_{t+s} G_{C,t+s}(i) \right] , \tag{217}$$

25 The presence of the growth terms ensures that adjustment costs are zero along the balanced growth path.

subject to demands (215) and adjustment costs (216). The first order condition for this problem, after dropping firm specific subscripts and rescaling by technology and population, has the form:

$$\left[\frac{\sigma_R - 1}{\sigma_R}\frac{p_t^R}{p_t^C} - 1\right] = \phi_C \left(\frac{\check{C}_t - \check{C}_{t-1}}{\check{C}_{t-1}}\right)\frac{\check{C}_t}{\check{C}_{t-1}} - E_t\frac{\theta g n}{\check{r}_{t+1}}\phi_C\left(\frac{\check{C}_{t+1} - \check{C}_t}{\check{C}_t}\right)\left(\frac{\check{C}_{t+1}}{\check{C}_t}\right)^2 . \quad (218)$$

The real **dividends** and rescaled adjustment costs of this sector are given by

$$\check{d}_t^R = (p_t^R - p_t^C)\check{C}_t - \check{G}_{C,t} , \quad (219)$$

$$\check{G}_{C,t} = \frac{\phi_C}{2}\check{C}_t\left(\frac{\check{C}_t - \check{C}_{t-1}}{\check{C}_{t-1}}\right)^2 . \quad (220)$$

When the retail sector is excluded from GIMF the foregoing simplifies to $p_t^R = p_t^C$.

15 Government

15.1 Government Production

The government uses consumption goods Y_t^{GC} and investment goods Y_t^{GI} to produce government output Z_t^G according to a CES production function with consumption goods share parameter α_{GC} and an elasticity of substitution ξ_G:

$$Z_t^G = \left((\alpha_{GC})^{\frac{1}{\xi_G}}\left(Y_t^{GC}\right)^{\frac{\xi_G - 1}{\xi_G}} + (1-\alpha_{GC})^{\frac{1}{\xi_G}}\left(Y_t^{GI}\right)^{\frac{\xi_G - 1}{\xi_G}}\right)^{\frac{\xi_G}{\xi_G - 1}} . \quad (221)$$

Denoting the marginal cost of producing Z_t^G by p_t^{ZG}, and normalizing by technology and population, we then obtain the normalized version of (221) and the following standard input demands:

$$\check{Y}_t^{GC} = \alpha_{GC}\check{Z}_t^G\left(p_t^{ZG}\right)^{\xi_G} , \quad (222)$$

$$\check{Y}_t^{GI} = (1-\alpha_{GC})\,\check{Z}_t^G\left(\frac{p_t^I}{p_t^{ZG}}\right)^{-\xi_G} . \quad (223)$$

We allow for unit root shocks to the relative price of government output. Specifically, the output of government goods $\check{Z}_t^G$ is converted to final output of government goods $\check{Y}_t^G$ using the technology

$$\check{Y}_t^G = T_t^G\check{Z}_t^G , \quad (224)$$

where T_t^G is a unit root technology shock with zero trend growth. We define the exogenous and stochastic relative price as $\tilde{p}_t^G = 1/T_t^G$. Then competitive pricing means that the final price of government output equals

$$p_t^G = \tilde{p}_t^G p_t^{ZG} \ . \tag{225}$$

Demand for government output $\check{G}_t$ comes from government consumption and investment:

$$\check{G}_t = \check{G}_t^{cons} + \check{G}_t^{inv} \ , \tag{226}$$

and the market clearing condition is given by $\check{G}_t = \check{Y}_t^G$, and therefore by

$$\check{Z}_t^G = \tilde{p}_t^G \check{G}_t \ . \tag{227}$$

15.2 Government Budget Constraint

Fiscal policy consists of a specification of public investment spending G_t^{inv}, public consumption spending G_t^{cons}, transfers from OLG agents to LIQ agents $\tau_{T,t} = \tau_{T,t}^{OLG} = \tau_{T,t}^{LIQ}$, lump-sum taxes $\tau_{ls,t} = \tau_t^{ls,OLG} + \tau_t^{ls,LIQ}$, lump-sum transfers $\Upsilon_t = \Upsilon_t^{OLG} + \Upsilon_t^{LIQ}$, and three different distortionary taxes $\tau_{L,t}$, $\tau_{c,t}$ and $\tau_{k,t}$.

Government investment and consumption spending $G_t = G_t^{inv} + G_t^{cons}$ represents a demand for government output. Both types of government spending are exogenous and stochastic. Government investment spending has a critical function in this economy. It augments the stock of publicly provided infrastructure capital K_t^{G1}, the evolution of which is, after rescaling by technology and population, given by

$$\check{K}_{t+1}^{G1} gn = (1 - \delta_{G1}) \check{K}_t^{G1} + \check{G}_t^{inv} \ , \tag{228}$$

where δ_{G1} is the depreciation rate of public capital. Government consumption spending on the other hand can be modeled as either unproductive or productive by choosing the coefficient α_{G2} in the production function. For the case of $\alpha_{G2} > 0$ government consumption accumulates a second productive capital stock:

$$\check{K}_{t+1}^{G2} gn = (1 - \delta_{G2}) \check{K}_t^{G2} + \check{G}_t^{cons} \ . \tag{229}$$

The government's policy rule for **transfers** partly compensates for the lack of asset ownership of LIQ agents by redistributing a small fraction of OLG agents's dividend income receipts to LIQ agents. Specifically, dividends of the retail and union sectors are redistributed in proportion to LIQ

agents' share in consumption and labor supply, while the redistributed share of dividends in the remaining sectors is ι, which we will typically calibrate as being smaller than the share ψ of LIQ agents in the population, $\iota = \psi d^{share}$ with $d^{share} < 1$. Finally, in the baseline of GIMF government lump-sum transfers and taxes are received and paid by LIQ agents in proportion to their share in aggregate consumption, but this rule can easily be changed, for example to allow for transfers that are 100% targeted to LIQ agents. After rescaling by technology we therefore have the following rule:

$$\begin{aligned} \check{\tau}_{T,t} &= \iota\left(\check{d}_t^N + \check{d}_t^T + \check{d}_t^D + \check{d}_t^C + \check{d}_t^I + \check{d}_t^M + \bar{d}^X + \check{d}_t^F + \check{d}_t^K + \check{d}_t^{EP}\right) \\ &\quad + \frac{\check{c}_t^{LIQ}}{\check{C}_t}\left(\check{d}_t^R + \check{\Upsilon}_t - \check{\tau}_t^{ls}\right) + \frac{\check{\ell}_t^{LIQ}}{\check{L}_t}\check{d}_t^U \ . \end{aligned} \quad (230)$$

The sources of nominal **tax revenue** are labor income taxes $\tau_{L,t} W_t L_t$, consumption taxes $\tau_{c,t} P_t^C C_t$, taxes on the return to capital $\tau_{k,t} \Sigma_{j=N,T} \left[R_{k,t}^J - \delta_{K_t}^J P_t q_t^J\right] K_t^J$, and lump-sum taxes $P_t \tau_{ls,t}$. We define the rescaled aggregate real tax variable for the case of GIMF without Financial Accelerator as

$$\check{\tau}_t = \tau_{L,t}\check{w}_t\check{L}_t + \tau_{c,t}p_t^C\check{C}_t + \check{\tau}_{ls,t} + \tau_{k,t}\Sigma_{j=N,T}\left[r_{k,t}^J - \delta_{K_t}^J q_t^J)\right]\frac{\overline{\check{K}}_{t-1}^J}{gn} \ , \quad (231)$$

while for GIMF with Financial Accelerator we have

$$\check{\tau}_t = \tau_{L,t}\check{w}_t\check{L}_t + \tau_{c,t}p_t^C\check{C}_t + \check{\tau}_{ls,t} + \tau_{k,t}\Sigma_{j=N,T}\left[u_t^J r_{k,t}^J - \delta_{K_t}^J q_t^J - a(u_t^J)\right]\frac{\overline{\check{K}}_{t-1}^J}{gn} \ . \quad (232)$$

Furthermore, the government issues nominally non-contingent one-period nominal debt B_t at the gross nominal interest rate i_t. The rescaled real **government budget constraint** is therefore

$$\check{b}_t + \check{\tau}_t + \check{g}_t^X = \frac{i_{t-1}}{\pi_t gn}\check{b}_{t-1} + p_t^G\check{G}_t + \check{\Upsilon}_t \ . \quad (233)$$

15.3 Fiscal Policy

The model makes two key assumptions about fiscal policy. The first concerns dynamic stability, and the second stabilization of the business cycle.

With respect to **dynamic stability**, fiscal policy ensures a non-explosive government debt to GDP ratio by adjusting tax rates to generate sufficient revenue, or by reducing expenditure, in order to stabilize the overall, interest inclusive government surplus to GDP ratio gs_t^{rat} at a long-run level of

gss_t^{rat} chosen by policy. This rules out partial default on government debt, and it also rules out fiscal dominance over monetary policy, implying that inflation will not be used as a tool of discretionary fiscal revenue generation. The government surplus is given by

$$gs_t = -\left(\check{b}_t - \frac{\check{b}_{t-1}}{\pi_t gn}\right) = \check{\tau}_t + \check{g}_t^X - p_t^G \check{G}_t - \check{\Upsilon}_t - \frac{i_{t-1} - 1}{\pi_t gn}\check{b}_{t-1} \,, \tag{234}$$

and its ratio to GDP (gdp_t will be defined below) is

$$gs_t^{rat} = -100\frac{B_t - B_{t-1}}{P_t gdp_t} = 100\frac{gs_t}{g\check{d}p_t} \,, \tag{235}$$

We allow for the possibility that gss_t^{rat} follows an exogenous stochastic process. We denote the current value and the long-run target for the government debt to GDP ratio by $\check{b}_t^{rat}$ and $\check{b}ss_t^{rat}$, expressed as a share of annual GDP. We have the following relationship between long-run government balance and government debt to GDP ratios:

$$gss_t^{rat} = -4\frac{\bar{\pi}_t gn - 1}{\bar{\pi}_t gn}\check{b}ss_t^{rat} \,. \tag{236}$$

Here $\bar{\pi}_t$ is the inflation target of the central bank. In other words, for a given nominal growth rate, choosing a surplus target gss_t^{rat} implies a debt target $\check{b}ss_t^{rat}$ and therefore keeps debt from exploding.

With respect to **business cycle stabilization**, fiscal policy ensures that the government surplus to GDP ratio, while satisfying its long-run target of gss_t^{rat}, can also flexibly respond to the business cycle. Specifically, we have the following structural fiscal surplus rule:

$$gs_t^{rat} = gss_t^{rat} + d^{debt}\left(\check{b}_t^{rat} - \check{b}ss_t^{rat}\right) + d^{tax}\left(\frac{\check{\tau}_t - \check{\tau}_t^{pot}}{g\check{d}p_t}\right) + d^{oil}\left(\frac{\check{g}_t^X - \check{g}_{X,t}^{pot}}{g\check{d}p_t}\right) \,. \tag{237}$$

The relationship (236) implies that even with $d^{debt} = 0$ the rule (237) automatically ensures a non-explosive government debt to GDP ratio of $\check{b}ss_t^{rat}$. But the long-run autoregressive coefficient on debt in that case, at $1/\left(\bar{\pi}_t gn\right)$, is very close to one. Setting $d^{debt} > 0$ ensures faster convergence of debt at the expense of more volatile government surpluses. The term $\check{\tau}_t^{pot}$ is tax revenue at current tax rates multiplied by the respective tax bases:

$$\check{\tau}_t^{pot} = \tau_{L,t} taxbase_{L,t}^{filt} + \tau_{C,t} taxbase_{C,t}^{filt} + \tau_{K,t} taxbase_{K,t}^{filt} + \bar{\tau}_{ls} \,. \tag{238}$$

Our model allows for unit root shocks to technology and to savings, where the latter have permanent real effects due to the non-Ricardian features of the model. The tax bases relevant for tax collection are therefore subject to nonstationary shocks. The tax revenue gap term in the fiscal rule has to reflect these changes, and the long-run tax bases in (238) are therefore formulated as moving averages of past (and if desired also future) actual tax bases. For applications of the model where unit root processes are not allowed for, these tax bases can simply be evaluated at their non-stochastic steady state. For the more general case, letting k_h^j, $j \in \{L, C, K\}$, be the maximum lead and k_l^j the maximum lag, we have[26]

$$taxbase_{L,t}^{filt} = E_t \left(\Sigma_{j=k_l^L}^{k_h^L} \check{w}_{t+j} \check{L}_{t+j} \right) / \left(k_h^L - k_l^L + 1 \right) \ , \tag{239}$$

$$taxbase_{C,t}^{filt} = E_t \left(\Sigma_{j=k_l^C}^{k_h^C} p_{t+j}^C \check{C}_{t+j} \right) / \left(k_h^C - k_l^C + 1 \right) \ , \tag{240}$$

$$taxbase_{K,t}^{filt} = \Sigma_{i \in N,T} E_t \left(\Sigma_{j=k_l^K}^{k_h^K} \left(u_{t+j}^i r_{k,t+j}^i - \delta_{K_{t+j}}^J q_{t+j}^i - a(u_{t+j}^i) \right) \frac{\overline{\check{K}}_{t+j-1}^i}{gn} \right) / \left(k_h^K - k_l^K + 1 \right) \ , \tag{241}$$

Setting $d^{tax} = 0$ in (237) corresponds to a balanced budget rule, which is highly procyclical and therefore undesirable. In a structural fiscal balance rule the assumption is $d^{tax} = 1$, so that during a boom, when tax revenue exceeds its long run value, the government uses the extra funds to pay off government debt by reducing the deficit below its long run value. The main effect is to minimize the variability of fiscal instruments, but of course it also reduces the variability of output relative to a balanced budget rule. A more explicitly counter-cyclical rule would set $d^{tax} > 1$.

As for potential raw materials revenue $\check{g}_{X_t}^{pot}$, we assume that it is based on estimates of the potential or long-run international price and domestic output of the raw material, which yields an estimate of potential dollar revenue. Changes in the real exchange rate are allowed to affect the estimate of potential revenue in terms of domestic currency:

$$g_{X_t}^{pot} = \left(e_t p_t^{X^*,filt} \check{X}_t^{sup,filt} - \bar{d}^X \right) \left(1 - s_f^x \right) \ , \tag{242}$$

where

$$p_t^{X^*,filt} = E_t \Sigma_{k=k_l^{px}}^{k_h^{px}} \left(p_{t+j}^{X^*} \right) / \left(k_h^{px} - k_l^{px} + 1 \right) \ , \tag{243}$$

$$\check{X}_t^{sup,filt} = E_t \Sigma_{k=k_l^{yx}}^{k_h^{yx}} \left(\check{X}_{t+j}^{sup} \right) / \left(k_h^{yx} - k_l^{yx} + 1 \right) \ . \tag{244}$$

[26] We only show $taxbase_{K,t}^{filt}$ for GIMF with Financial Accelerator. The alternative follows trivially by setting $u_{t+j}^i \equiv 1$. Future versions of GIMF will add variable capital utilization to the version without Financial Accelerator.

The rule (237) is not an instrument rule but rather a targeting rule. Any of the available tax and spending instruments can be used to make sure the rule holds. The default setting is that this instrument is the labor tax rate $\tau_{L,t}$, because this is the most plausible choice. However, other instruments or combinations of multiple instruments are possible. For example, we can posit

$$\tau_{c,t} = \bar{\tau}_c + d^{ctax}\left(\tau_{L,t} - \bar{\tau}_L\right) \ , \tag{245}$$

$$\tau_{k,t} = \bar{\tau}_k + d^{ktax}\left(\tau_{L,t} - \bar{\tau}_L\right) \ . \tag{246}$$

With $d^{ctax} = d^{ktax} = 1$ this generates a perfect comovement between the three tax rates, while $d^{ctax} = d^{ktax} = 0$ means that only labor tax rates change.

15.4 Monetary Policy

Monetary policy uses an interest rate rule that features interest rate smoothing and which responds to (i) deviations of one-year-ahead year-on-year inflation $\pi_{4,t+4}$ from the stochastic inflation target $\bar{\pi}_t$, (ii) the output gap, using Fisher-weighted GDP $g\check{d}p_t^{fisher}$ as the relevant output measure, (iii) the year-on-year growth rate of Fisher-weighted GDP, and (iv) deviations of current exchange rate depreciation from its long run value $\bar{\varepsilon}_t = \bar{\pi}_t/\bar{\pi}_t^*$. Furthermore, we allow for autocorrelated monetary policy shocks S_t^{int}. The rule is very general and similar to the class of rules suggested by Orphanides (2003), with one minor and one important exception. The minor exception is the presence of exchange rate depreciation, which we will however only use for the case of strict exchange rate targeting, which can be modeled as $\delta_i = 1$ and $\delta_e \longrightarrow \infty$. The important exception is that the non-Ricardian nature of the model implies that there is no unchanging steady state real interest rate or GDP. Instead the long run real interest rate and GDP are determined by permanent shocks, including to public and private savings preferences. The term proxying the long run value of GDP in the output gap term therefore includes an arithmetic moving average of GDP similar to the tax base averages above. And the term proxying the nominal interest rate $r_t^{filt}\pi_{4,t+4}$ includes a geometric moving average of real interest rates, but this average is more complicated. Specifically, it contains separate moving averages of the underlying pre-risk-premium real interest rate, r_t^{world}, and of the risk premium itself, ξ_t^{filt}. As for the former, in order to exclude excessive recent fluctuations in the domestic real interest rate from the proxy of the underlying equilibrium real interest rate, we include a smoothed measure of a worldwide GDP-weighted average real interest rate. The separate smoothing of the risk premium terms is done in the usual way and multiplies r_t^{world}. We adopt the notation $r_t^{pre\xi} = r_t / \left(\left(1 + \xi_t^f \right) \left(1 + \xi_t^b \right) \right)$ and $\xi_t = \left(1 + \xi_t^f \right) \left(1 + \xi_t^b \right)$. Then the complete monetary rule is given by

$$\begin{aligned} i_t \;\; = \;\; & E_t \left(i_{t-1} \right)^{\delta_i} \left(r_t^{filt} \pi_{4,t+4} \right)^{1-\delta_i} \left(\frac{\pi_{4,t+4}}{\bar{\pi}_t} \right)^{(1-\delta_i)\delta_\pi} \\ & \left(\frac{g\check{d}p_t^{fisher}}{g\check{d}p_t^{filt}} \right)^{(1-\delta_i)\delta_y} \left[\left(\frac{g\check{d}p_t^{fisher}}{g\check{d}p_{t-4}^{fisher}} \right) \right]^{(1-\delta_i)\delta_{ygr}} \left(\frac{\varepsilon_t}{\bar{\varepsilon}_t} \right)^{\delta_e} S_t^{int} \; , \end{aligned} \tag{247}$$

$$\pi_{4,t} = \left(\pi_t \pi_{t-1} \pi_{t-2} \pi_{t-3} \right)^{\frac{1}{4}} \; , \tag{248}$$

$$r_t^{filt} = r_t^{world} \xi_t^{filt} \; , \tag{249}$$

$$r_t^{world} = \Pi_{j=1}^{\tilde{N}} \left(r_t^{smooth(j)} \right)^{\frac{gdp_{ss}(j)}{\Sigma_{i=1}^{\tilde{N}} gdp_{ss}(i)}} \; , \tag{250}$$

$$r_t^{smooth} = E_t \left(\Pi_{j=k_l^r}^{k_h^r} r_{t+j}^{pre\xi} \right)^{\frac{1}{k_h^r - k_l^r + 1}} \; , \tag{251}$$

$$\xi_t^{filt} = E_t \left(\Pi_{j=k_l^r}^{k_h^r} \xi_{t+j} \right)^{\frac{1}{k_h^r - k_l^r + 1}} \; , \tag{252}$$

$$g\check{d}p_t^{filt} = E_t \left(\Sigma_{j=k_l^{gdp}}^{k_h^{gdp}} g\check{d}p_{t+j}^{fisher} \right) / \left(k_h^{gdp} - k_l^{gdp} + 1 \right) \; . \tag{253}$$

16 Shocks

We assume that β_t, $\alpha_{H_t}^C$, $\alpha_{H_t}^I$, $\alpha_{H_t}^T$, $\alpha_{C_t}^X$, $\alpha_{N_t}^X$, $\alpha_{T_t}^X$, X_t^{sup}, σ_t^N, σ_t^T, μ_t^N, μ_t^T, $\check{S}_t^{N,nwd}$, $\check{S}_t^{T,nwd}$, $\check{S}_t^{N,nwy}$, $\check{S}_t^{T,nwy}$, $\check{S}_t^{N,nwk}$, $\check{S}_t^{T,nwk}$, $\check{G}_t^{cons}$ and $\check{G}_t^{inv}$, and their foreign counterparts, are characterized by both transitory and unit root components. Denoting any of these shocks by x_t we have

$$x_t = \left(1 - \rho_x \right) \tilde{x}_t + \rho_x x_{t-1} + u_t^x \tilde{x}_t \; , \tag{254}$$

$$\ln(\tilde{x}_t) = \ln(\tilde{x}_{t-1}) + u_t^{\tilde{x}} \; . \tag{255}$$

For the two policy variables gss_t^{rat} and $\bar{\pi}_t$ the transitory components are given by the endogenous responses of the fiscal and monetary rules, while the permanent components are specified as unit roots:

$$\ln(\bar{\pi}_t) = \ln(\bar{\pi}_{t-1}) + u_t^{\pi} \; , \tag{256}$$

$$gss_t^{rat} = gss_{t-1}^{rat} + u_t^{gss} \quad . \tag{257}$$

For the three relative price processes $\tilde{p}_t^y, y \in \{I, G, exp\}$ we also assume unit roots:

$$\ln(\tilde{p}_t^y) = \ln(\tilde{p}_{t-1}^y) + u_t^{py} \quad , \tag{258}$$

Interest rate, investment, labor supply, foreign exchange risk premium, government risk premium and markup shocks are assumed to only have transitory components, and markup shocks in addition are assumed to be serially uncorrelated:[27]

$$S_t^{int} = (1 - \rho_{int}) + \rho_{int} S_{t-1}^{int} + u_t^{int} \quad , \tag{259}$$

$$S_t^{inv} = (1 - \rho_{inv}) + \rho_{inv} S_{t-1}^{inv} + u_t^{inv} \quad , \tag{260}$$

$$S_t^{L} = (1 - \rho_{L}) + \rho_{L} S_{t-1}^{L} + u_t^{L} \quad , \tag{261}$$

$$\xi_t^f = \rho_{fxp} \xi_{t-1}^f + u_t^{fxp} \quad , \tag{262}$$

$$\xi_t^b = \rho_{gbp} \xi_{t-1}^b + u_t^{gbp} \quad , \tag{263}$$

$$\mu_t^i = \bar{\mu}^i \left(1 + u_t^{\mu^i}\right) \quad , \quad i = U, C, I \ . \tag{264}$$

For productivity shocks, we allow country specific technology to follow the U.S., in the following way:

$$\text{US: } A_t^{J(US)} = (1 - \rho^{A^J(US)} + \boldsymbol{e}_t^{A^J(US)}) \tilde{A}_t^{J(US)} + \rho^{A^J(US)} A_{t-1}^{J(US)} \ , \tag{265}$$

$$\begin{aligned} \text{Country } j \quad : \qquad A_t^{J(j)} &= (1 - \rho^{A^J(j)}) \left(\tilde{A}_t^{J(j)} + catchup(j) * \left(A_t^{J(US)} - \tilde{A}_t^{J(US)}\right)\right) \\ &+ \rho^{A^J(j)} A_{t-1}^{J(j)} + \boldsymbol{e}_t^{A^J(j)} \tilde{A}_t^{J(j)} \ . \end{aligned} \tag{266}$$

The parameter $catchup(j)$ can vary between 0 and 1, and $\tilde{A}_t^J$ can be subject to unit root shocks. For the stationary shock to the price of investment goods we again allow for catchup growth with the U.S.:

$$\text{US: } \breve{p}_t^{I(US)} = (1 - \rho^{pi(US)} + \boldsymbol{e}_t^{pi(US)}) + \rho^{pi(US)} \breve{p}_{t-1}^{I(US)} \ , \tag{267}$$

$$\text{Country } j\text{: } \breve{p}_t^{I(j)} = (1 - \rho^{pi(j)}) \left(1 + catchup(j) * \left(\breve{p}_t^{I(US)} - 1\right)\right) + \rho^{pi(j)} \breve{p}_{t-1}^{I(j)} + \boldsymbol{e}_t^{pi(j)} \ . \tag{268}$$

[27] Inflation persistence in the model is therefore exclusively due to inflation adjustment costs.

17 Balance of Payments

Combining all market clearing conditions with the budget constraints of households and the government and with the expressions for firm dividends we obtain an expression for the current account:

$$
\begin{aligned}
e_t \check{f}_t &= \frac{i_{t-1}(\tilde{N})\varepsilon_t(1+\xi^f_{t-1})}{\pi_t g n} e_{t-1}\check{f}_{t-1} \\
&\quad + p^{TH}_t \tilde{p}^{exp}_t \Sigma^{\tilde{N}}_{j=2} \check{Y}^{TX}_t(1,j) + \check{d}^{TM}_t - p^{TF}_t \check{Y}^{TF}_t \\
&\quad + p^{DH}_t \tilde{p}^{exp}_t \Sigma^{\tilde{N}}_{j=2} \check{Y}^{DX}_t(1,j) + \check{d}^{DM}_t - p^{DF}_t \check{Y}^{DF}_t \\
&\quad + \check{X}^x_t - \check{f}^X_t \; .
\end{aligned}
\tag{269}
$$

When we repeat the same exercise for all other countries we finally obtain the market clearing condition for international bonds,

$$
\Sigma^{\tilde{N}}_{j=1} \check{f}_t(j) = 0 \; . \tag{270}
$$

The current account balance is given by

$$
ca_t = e_t \check{f}_t - \frac{e_{t-1}\check{f}_{t-1}}{\pi_t g n} \; . \tag{271}
$$

The level of GDP is given by the following expression:

$$
\begin{aligned}
g\check{d}p_t &= p^C_t \check{C}_t + p^I_t \check{I}_t + p^G_t \check{G}_t + \check{X}^x_t \\
&\quad + p^{TH}_t \tilde{p}^{exp}_t \Sigma^{\tilde{N}}_{j=2} \check{Y}^{TX}_t(1,j) + \check{d}^{TM}_t - p^{TF}_t \check{Y}^{TF}_t \\
&\quad + p^{DH}_t \tilde{p}^{exp}_t \Sigma^{\tilde{N}}_{j=2} \check{Y}^{DX}_t(1,j) + \check{d}^{DM}_t - p^{DF}_t \check{Y}^{DF}_t \; .
\end{aligned}
\tag{272}
$$

Appendix A. Population Growth

The population size at time 0 is assumed to equal N, with $N(1-\psi)$ OLG households and $N\psi$ LIQ households. The size of a new cohort born at time t is given by $Nn^t\left(1-\frac{\theta}{n}\right)$, so that by time $t+k$ this cohort will be of size $Nn^t\left(1-\frac{\theta}{n}\right)\theta^k$. When we sum over all cohorts at time t we obtain

$$
\begin{aligned}
& Nn^t\left(1-\frac{\theta}{n}\right)+Nn^{t-1}\left(1-\frac{\theta}{n}\right)\theta+Nn^{t-2}\left(1-\frac{\theta}{n}\right)\theta^2+... \\
= \; & Nn^t\left(1-\frac{\theta}{n}\right)\left(1+\frac{\theta}{n}+\left(\frac{\theta}{n}\right)^2+...\right) \\
= \; & Nn^t \; .
\end{aligned}
$$

This means that the overall population grows at the rate n. When we normalize real quantities, we divide by the level of technology T_t and by population, but for the latter we divide by n^t only, meaning real figures are not in per capita terms but rather in absolute terms adjusted for population growth.

Appendix B. Optimality Conditions for OLG Households

We have the following Lagrangian representation of the optimization problem of OLG households:[28]

$$
\mathcal{L}_{a,t}=E_t\sum_{s=0}^{\infty}(\beta\theta)^s\left\{\left[\frac{1}{1-\gamma}\left(\left(c^{OLG}_{a+s,t+s}\right)^{\eta^{OLG}}\left(S^L_t-\ell^{OLG}_{a+s,t+s}\right)^{1-\eta^{OLG}}\right)^{1-\gamma}\right]\right\} \tag{273}
$$

$$
\begin{aligned}
&+\Lambda_{a+s,t+s}\left[\frac{1}{\theta}\left[\frac{i_{t-1+s}}{\left(1+\xi^b_{t-1+s}\right)}\left(B_{a-1+s,t-1+s}+B^N_{a-1+s,t-1+s}+B^T_{a-1+s,t-1+s}\right)+i^*_{t-1+s}\mathcal{E}_{t+s}F_{a-1+s,t-1+s}(1+\xi^f_{t-1})\right.\right. \\
&\quad +W_{t+s}\Phi_{a+s,t+s}\ell^{OLG}_{a+s,t+s}(1-\tau_{L,t+s})+\sum_{j=N,T,D,C,I,R,U,M,X,F,K,EP}\int_0^1 D^j_{a+s,t+s}(i)di-P_t\tau^{OLG}_{T_{a,t}} \\
&\quad -\left[P_{t+s}c^{OLG}_{a+s,t+s}(p^R_{t+s}+p^C_{t+s}\tau_{c,t+s})+B_{a+s,t+s}+B^N_{a+s,t+s}+B^T_{a+s,t+s}+\mathcal{E}_{t+s}F_{a+s,t+s}\right]\Big\} \; ,
\end{aligned}
$$

where $\Lambda_{a,t}$ is the marginal utility to the generation of age a at time t of an extra unit of domestic currency. Define the marginal utility of an extra unit of consumption goods output as

$$
\lambda_{a,t}=\Lambda_{a,t}P_t \; , \tag{274}
$$

[28] For simplicity we ignore money given the cashless limit assumption.

and let

$$u_{a,t}^{OLG} = \left(c_{a,t}^{OLG}\right)^{\eta^{OLG}} \left(S_t^L - \ell_{a,t}^{OLG}\right)^{1-\eta^{OLG}} \quad . \tag{275}$$

Then we have the following first-order conditions for consumption and labor supply

$$\frac{\eta^{OLG}\left(u_{a,t}^{OLG}\right)^{1-\gamma}}{c_{a,t}^{OLG}} = \lambda_{a,t}(p_t^R + p_t^C \tau_{c,t}) \ , \tag{276}$$

$$\frac{\left(1-\eta^{OLG}\right)\left(u_{a,t}^{OLG}\right)^{1-\gamma}}{S_t^L - \ell_{a,t}^{OLG}} = \lambda_{a,t} w_t \Phi_{a,t}(1-\tau_{L,t}) \ , \tag{277}$$

which can be combined to yield

$$\frac{c_{a,t}^{OLG}}{1-\ell_{a,t}^{OLG}} = \frac{\eta^{OLG}}{1-\eta^{OLG}} w_t \Phi_{a,t} \frac{(1-\tau_{L,t})}{(p_t^R + p_t^C \tau_{c,t})} \ . \tag{278}$$

We can aggregate this as

$$\frac{c_t^{OLG}}{Nn^t(1-\psi)S_t^L - \ell_t^{OLG}} = \frac{\eta^{OLG}}{1-\eta^{OLG}} w_t \frac{(1-\tau_{L,t})}{(p_t^R + p_t^C \tau_{c,t})} \ , \tag{279}$$

and normalize it as

$$\frac{\check{c}_t^{OLG}}{N(1-\psi)S_t^L - \check{\ell}_t^{OLG}} = \frac{\eta^{OLG}}{1-\eta^{OLG}} \check{w}_t \frac{(1-\tau_{L,t})}{(p_t^R + p_t^C \tau_{c,t})} \ . \tag{280}$$

In this aggregation we have made use of the following assumptions about labor productivity:

$$\Phi_{a,t} = \kappa \chi^a \ , \tag{281}$$

$$Nn^t(1-\psi)\left(1-\frac{\theta}{n}\right)\Sigma_{a=0}^{\infty}\left(\frac{\theta}{n}\right)^a \Phi_{a,t} = Nn^t(1-\psi) \ , \tag{282}$$

$$\kappa = \frac{(n-\theta\chi)}{(n-\theta)} \ , \tag{283}$$

$$Nn^t(1-\psi)\left(1-\frac{\theta}{n}\right)\Sigma_{a=0}^{\infty}\left(\frac{\theta}{n}\right)^a \left(\ell_{a,t}^{OLG}\Phi_{a,t}\right) \equiv \ell_t^{OLG} \ . \tag{284}$$

Equation (281) is our specification of the profile of labor productivity over the lifetime. Equation (282) is the assumption that average labor productivity equals one. Equations (281) and (282), for a given productivity decline parameter χ, imply the initial productivity level κ in (283). Equation (284) is the definition of effective aggregate labor supply.

Next we have the first-order conditions for domestic and foreign bonds $B_{a,t}$ and $F_{a,t}$:

$$\lambda_{a,t} = \beta E_t \lambda_{a+1,t+1} \frac{i_t}{\pi_{t+1}(1+\xi_t^b)} \ , \tag{285}$$

$$\lambda_{a,t} = \beta E_t \lambda_{a,t+1} \frac{i_t^* \varepsilon_{t+1}(1+\xi_t^f)}{\pi_{t+1}} \ . \tag{286}$$

Together these yield the uncovered interest parity condition

$$i_t = i_t^* \tilde{E}_t \varepsilon_{t+1}(1+\xi_t^f)(1+\xi_t^b) \ . \tag{287}$$

To write the marginal utility of consumption $\lambda_{a,t}$ in terms of quantities that can be aggregated, specifically in terms of consumption, we use (275) and (278) in (276) to get

$$\lambda_{a,t} = \eta^{OLG} \left(c_{a,t}^{OLG}\right)^{-\gamma} \left(p_t^R + p_t^C \tau_{c,t}\right)^{-1} \left(\frac{(1-\eta^{OLG})(p_t^R + p_t^C \tau_{c,t})}{\eta^{OLG} w_t \Phi_{a,t}(1-\tau_{L,t})}\right)^{(1-\eta^{OLG})(1-\gamma)} \ . \tag{288}$$

We use (288) in (285) to obtain the generation specific consumption Euler equations

$$\tilde{E}_t c_{a+1,t+1}^{OLG} = \tilde{E}_t j_t c_{a,t}^{OLG} \ , \text{ where} \tag{289}$$

$$j_t = \left(\frac{\beta i_t}{\pi_{t+1}(1+\xi_t^b)}\right)^{\frac{1}{\gamma}} \left(\frac{p_t^R + p_t^C \tau_{c,t}}{p_{t+1}^R + p_{t+1}^C \tau_{c,t+1}}\right)^{\frac{1}{\gamma}} \left(\chi g \frac{\breve{w}_{t+1}(1-\tau_{L,t+1})(p_t^R + p_t^C \tau_{c,t})}{\breve{w}_t(1-\tau_{L,t})(p_{t+1}^R + p_{t+1}^C \tau_{c,t+1})}\right)^{(1-\eta^{OLG})(1-\frac{1}{\gamma})} \ . \tag{290}$$

Appendix C. Consumption and Wealth

The key equation for OLG households is the one relating current consumption to current wealth. We start deriving this by reproducing the budget constraint:

$$P_t c_{a,t}^{OLG}(p_t^R + p_t^C \tau_{c,t}) + B_{a,t} + B_{a,t}^N + B_{a,t}^T + \mathcal{E}_t F_{a,t} \tag{291}$$

$$= \frac{1}{\theta}\left[\frac{i_{t-1}}{(1+\xi_{t-1}^b)}\left(B_{a-1,t-1} + B_{a-1,t-1}^N + B_{a-1,t-1}^T\right) + i_{t-1}^* \mathcal{E}_t F_{a-1,t-1}(1+\xi_{t-1}^f)\right]$$

$$+ W_t \Phi_{a,t} \ell_{a,t}^{OLG}(1-\tau_{L,t}) + \sum_{j=N,T,D,C,I,R,U,M,X,F,K,EP} \int_0^1 D_{a,t}^j(i)di + P_t\left(\Upsilon_{a,t} - \tau_{a,t}^{ls}\right) - P_t \tau_{T_{a,t}}^{OLG} \ .$$

We now derive an expression that decomposes human wealth into labor and dividend income. First, we note that after-tax wage income can be decomposed as follows:

$$W_t \Phi_{a,t} \ell_{a,t}^{OLG}(1-\tau_{L,t}) = W_t \Phi_{a,t}(1-\tau_{L,t}) S_t^L - W_t \Phi_{a,t}(1-\tau_{L,t})(S_t^L - \ell_{a,t}^{OLG}) \ . \tag{292}$$

The first expression on the right-hand side of (292) is the labor component of income, which equals the marginal value of the household's entire endowment (one unit) of time. The second expression in (292), by (278), can be rewritten as

$$W_t\Phi_{a,t}(1-\tau_{L,t})(S_t^L-\ell_{a,t}^{OLG})=\frac{1-\eta^{OLG}}{\eta^{OLG}}P_tc_{a,t}^{OLG}(p_t^R+p_t^C\tau_{c,t})\ , \tag{293}$$

which can be combined with the consumption expression in (291) to obtain, on the left-hand side of (291), $P_tc_{a,t}^{OLG}(p_t^R+p_t^C\tau_{c,t})/\eta^{OLG}$. The second component of income is dividend and net transfer income net of redistribution to LIQ agents, the expression for which can be simplified by noting that in equilibrium all firms in a given sector pay equal dividends, so that we can drop the firm specific index and write $\int_0^1 D_{a,t}^j(i)di = D_{a,t}^j$. We also assume that per capita dividends and net transfers received by each OLG agent are identical. Finally, we incorporate the assumption that a share of dividend and net transfer income is redistributed to LIQ agents:

$$P_t\tau_{T_{a,t}}^{OLG}=\iota\sum_{j=N,T,D,C,I,M,X,F,K,EP}D_{a,t}^j+\frac{\check{c}_t^{LIQ}}{\check{C}_t}\left(D_{a,t}^R+P_t\Upsilon_{a,t}-P_t\tau_{a,t}^{ls}\right)+\frac{\check{\ell}_t^{LIQ}}{\check{L}_t}D_{a,t}^U\ . \tag{294}$$

These assumptions imply

$$\sum_{j=N,T,D,C,I,R,U,M,X,F,K,EP}\int_0^1 D_{a,t}^j(i)di-P_t\tau_{T_{a,t}}^{OLG} \tag{295}$$

$$=\sum_{j=N,T,D,C,I,M,X,F,K,EP}\frac{D_t^j(1-\iota)}{Nn^t(1-\psi)}+\frac{\check{c}_t^{OLG}}{\check{C}_t}\frac{\left(D_t^R+P_t\Upsilon_t-P_t\tau_t^{ls}\right)}{Nn^t(1-\psi)}+\frac{\check{\ell}_t^{OLG}}{\check{L}_t}\frac{D_t^U}{Nn^t(1-\psi)}\ .$$

The preceding arguments imply that total nominal wage and dividend income of households of age a in period t is given by

$$\begin{aligned}Inc_{a,t} &= W_t\Phi_{a,t}(1-\tau_{L,t})S_t^L \\ &\quad+\sum_{j=N,T,D,C,I,M,X,F}\frac{D_t^j(1-\iota)}{Nn^t(1-\psi)}+\frac{\check{c}_t^{OLG}}{\check{C}_t}\frac{\left(D_t^R+P_t\Upsilon_t-P_t\tau_t^{ls}\right)}{Nn^t(1-\psi)}+\frac{\check{\ell}_t^{OLG}}{\check{L}_t}\frac{D_t^U}{Nn^t(1-\psi)}\ .\end{aligned} \tag{296}$$

We now rewrite the household budget constraint as follows:

$$P_tc_{a,t}^{OLG}\frac{(p_t^R+p_t^C\tau_{c,t})}{\eta^{OLG}}+B_{a,t}+B_{a,t}^N+B_{a,t}^T+\mathcal{E}_tF_{a,t} \tag{297}$$

$$=Inc_{a,t}+\frac{1}{\theta}\left[\frac{i_{t-1}}{(1+\xi_{t-1}^b)}\left(B_{a-1,t-1}+B_{a-1,t-1}^N+B_{a-1,t-1}^T\right)+i_{t-1}^*\mathcal{E}_tF_{a-1,t-1}(1+\xi_{t-1}^f)\right]\ .$$

We proceed to derive a condition relating current consumption to lifetime wealth through successive forward substitutions of (297). In doing so we use the arbitrage condition (286) to cancel terms relating to foreign bonds. After the first substitution we obtain

$$\frac{\theta(1+\xi_t^b)}{i_t}\tilde{E}_t\left\{B_{a+1,t+1}+B_{a+1,t+1}^N+B_{a+1,t+1}^T+\mathcal{E}_{t+1}F_{a+1,t+1}\right\} \tag{298}$$

$$+P_t c_{a,t}^{OLG}\frac{(p_t^R+p_t^C\tau_{c,t})}{\eta^{OLG}}+\frac{\theta(1+\xi_t^b)}{i_t}\tilde{E}_t\left\{P_{t+1}c_{a+1,t+1}^{OLG}\frac{(p_{t+1}^R+p_{t+1}^C\tau_{c,t+1})}{\eta^{OLG}}\right\}=$$

$$Inc_{a,t}+\frac{\theta(1+\xi_t^b)}{i_t}\tilde{E}_t\left\{Inc_{a+1,t+1}\right\}+\frac{1}{\theta}\left[\frac{i_{t-1}}{(1+\xi_{t-1}^b)}\left(B_{a-1,t-1}+B_{a-1,t-1}^N+B_{a-1,t-1}^T\right)+i_{t-1}^*\mathcal{E}_tF_{a-1,t-1}(1+\xi_{t-1}^f)\right]$$

and successively substitute forward in the same fashion. We impose the following no-Ponzi condition on the household's optimization problem:

$$\lim_{s\longrightarrow\infty}\tilde{E}_t\tilde{R}_{t,s}\left[B_{a+s,t+s}+B_{a+s,t+s}^N+B_{a+s,t+s}^T+\mathcal{E}_{t+s}F_{a+s,t+s}\right]=0\ . \tag{299}$$

Furthermore, we let

$$FW_{a-1,t-1}=\frac{1}{\theta}\left[\frac{i_{t-1}}{(1+\xi_{t-1}^b)}\left(B_{a-1,t-1}+B_{a-1,t-1}^N+B_{a-1,t-1}^T\right)+i_{t-1}^*\mathcal{E}_tF_{a-1,t-1}(1+\xi_{t-1}^f)\right]\ . \tag{300}$$

This expression denotes nominal financial wealth inherited from period $t-1$. Next we define a variable $HW_{a,t}$ denoting lifetime human wealth, which equals the present discounted value of future incomes Inc_t. We have

$$HW_{a,t}=\tilde{E}_t\Sigma_{s=0}^{\infty}\tilde{R}_{t,s}Inc_{a+s,t+s}\ . \tag{301}$$

Further forward substitutions on (298), and application of the transversality condition (299), then yields the following:

$$\tilde{E}_t\Sigma_{s=0}^{\infty}\tilde{R}_{t,s}\left[P_{t+s}c_{a+s,t+s}^{OLG}\frac{(p_{t+s}^R+p_{t+s}^C\tau_{c,t+s})}{\eta^{OLG}}\right]=HW_{a,t}+FW_{a-1,t-1}\ . \tag{302}$$

The left-hand side of this expression can be further evaluated by using (289) for all future consumption terms. We let

$$\begin{aligned} j_{t,s} &= 1 && \text{for } s=0, \\ &= \Pi_{l=1}^{s}j_{t+l-1} && \text{for } s\geq 1\ . \end{aligned} \tag{303}$$

Then we can write

$$P_t c_{a,t}^{OLG} \tilde{E}_t \left(\Sigma_{s=0}^{\infty} \tilde{r}_{t,s} j_{t,s} \frac{(p_{t+s}^R + p_{t+s}^C \tau_{c,t+s})}{\eta^{OLG}} \right) = HW_{a,t} + FW_{a-1,t-1} \ . \tag{304}$$

The infinite summation on the left-hand side is recursive and can be written as

$$\Theta_t = \tilde{E}_t \Sigma_{s=0}^{\infty} \tilde{r}_{t,s} j_{t,s} \frac{(p_{t+s}^R + p_{t+s}^C \tau_{c,t+s})}{\eta^{OLG}} = \frac{(p_t^R + p_t^C \tau_{c,t})}{\eta^{OLG}} + \tilde{E}_t \frac{\theta j_t}{\check{r}_t} \Theta_{t+1} \ , \tag{305}$$

so we finally obtain

$$P_t c_{a,t}^{OLG} \Theta_t = HW_{a,t} + FW_{a-1,t-1} \ . \tag{306}$$

We want to express this equation in real aggregate terms. We begin with real aggregate human wealth, denoted by hw_t:

$$hw_t = N n^t (1-\psi) \left(1 - \frac{\theta}{n}\right) \Sigma_{a=0}^{\infty} \left(\frac{\theta}{n}\right)^a \frac{HW_{a,t}}{P_t} \ . \tag{307}$$

We break this down into its labor income and dividend income components hw_t^L and hw_t^K. For hw_t^L we have

$$hw_t^L = \tilde{E}_t \Sigma_{s=0}^{\infty} \tilde{r}_{t,s} \chi^s \left(N n^t (1-\psi) w_{t+s} (1 - \tau_{L,t+s}) S_{t+s}^L \right) \ ,$$

where we have used (281) and (283). In recursive form, and scaling by technology, the last equation equals

$$\check{h}w_t^L = \left(N(1-\psi) \check{w}_t (1 - \tau_{L,t}) S_t^L \right) + \tilde{E}_t \frac{\theta \chi g}{\check{r}_t} \check{h}w_{t+1}^L \ . \tag{308}$$

For hw_t^K we have, using (295) and letting $d_t^j = D_t^j / P_t$,

$$hw_t^K = \tilde{E}_t \Sigma_{s=0}^{\infty} \tilde{r}_{t,s} \left(\Sigma_{j=N,T,D,C,I,M,X,F,K,EP} d_t^j (1-\iota) + \frac{\check{c}_t^{OLG}}{\check{C}_t} \left(d_t^R + \Upsilon_t - \tau_t^{ls} \right) + \frac{\check{\ell}_t^{OLG}}{\check{L}_t} d_t^U \right) \ ,$$

which has the recursive representation, again after scaling by technology, of

$$\check{h}w_t^K = \left(\Sigma_{j=N,T,D,C,I,M,X,F,K,EP} \check{d}_t^j (1-\iota) + \frac{\check{c}_t^{OLG}}{\check{C}_t} \left(\check{d}_t^R + \check{\Upsilon}_t - \check{\tau}_t^{ls} \right) + \frac{\check{\ell}_t^{OLG}}{\check{L}_t} \check{d}_t^U \right) + \tilde{E}_t \frac{\theta g}{\check{r}_t} \check{h}w_{t+1}^K \ . \tag{309}$$

Finally, we have

$$\check{h}w_t = \check{h}w_t^L + \check{h}w_t^K \ . \tag{310}$$

Next we aggregate over the financial wealth of different age groups. We note here that aggregation cancels the $1/\theta$ term in front of the bracket in (300). This is because the period by period budget constraint (291) from which (300) was derived is the budget constraint of the agents that have in fact survived from period $t-1$ to t. Aggregation has to take account of the fact that $(1-\theta)$ agents did not survive and their wealth passed, through the insurance company, to surviving agents. Noting that $B_{-1,t-1} = 0$, we therefore have[29]

$$B_{t-1} = Nn^t(1-\psi)\left(1-\frac{\theta}{n}\right)\Sigma_{a=0}^{\infty}\left(\frac{\theta}{n}\right)^{a-1} B_{a-1,t-1} \ .$$

For total nominal financial wealth, we therefore have

$$FW_{t-1} = \left[\frac{i_{t-1}}{(1+\xi_{t-1}^b)}\left(B_{t-1} + B_{t-1}^N + B_{t-1}^T\right) + i_{t-1}^* \mathcal{E}_t F_{t-1}(1+\xi_{t-1}^f)\right] \ .$$

To express this in real terms, we define the real domestic currency asset stock as $b_t = B_t/P_t$. We adopt the convention that each nominal asset is deflated by the consumption based price index of the currency of its denomination, so that $f_t = F_t/P_t^*$. With the real exchange rate in terms of final output denoted by $e_t = \mathcal{E}_t P_t^*/P_t$, and after scaling by technology and population, we can then write

$$\check{f}w_t = \frac{FW_{t-1}}{P_t T_t n^t} = \frac{1}{\pi_t g n}\left[\frac{i_{t-1}}{(1+\xi_{t-1}^b)}\left(\check{b}_{t-1} + \check{b}_{t-1}^N + \check{b}_{t-1}^T\right) + i_{t-1}^* \varepsilon_t \check{f}_{t-1} e_{t-1}(1+\xi_{t-1}^f)\right] \ . \quad (311)$$

Finally, using (306)-(311) we arrive at our final expression for current period consumption:

$$\check{c}_t^{OLG}\Theta_t = \check{h}w_t + \check{f}w_t \ . \quad (312)$$

The linearized form of the aggregate equation (312) can instead be derived by linearizing an individual age group's budget constraint, using its linearized optimality conditions, and then aggregating over all generations. As mentioned above, it is therefore appropriate to use the expectations operator $\tilde{E}_t$ in nonlinear equations as long as it is understood that this is valid only up to first-order approximations of the system.

[29] Take the example of bonds held by those of age 0 at time $t-1$. Only θ of those agents survive into period t, but those that do survive obtain $1/\theta$ units of currency for every unit they held in $t-1$. Their weight in period t bonds aggregation is therefore $\theta\frac{1}{\theta} = 1$.

Appendix D. Manufacturers - Without Financial Accelerator

The objective function facing each manufacturing firm in sectors $J \in \{N, T\}$ is

$$\underset{P_s^J(i), U_s^J(i), I_s^J(i), K_{s+1}^J(i)}{Max} E_t \Sigma_{s=t}^{\infty} \tilde{R}_{t,s} D_{t+s}^J(i) \ .$$

The price (and inflation) terms in the two sectors will be indexed with $\tilde{J} \in \{N, TH\}$. Then dividend terms are given by

$$\begin{aligned} D_t^J(i) \ = \ & P_t^{\tilde{J}}(i) Z_t^J(i) - V_t U_t^J(i) - P_t^X X_t^J(i) - P_t^I I_t^J(i) \\ & - V_t G_{U,t}^J(i) - P_t^I G_{I,t}^J(i) - P_t^{\tilde{J}} G_{P,t}^J(i) - P_t^{\tilde{J}} T_t \omega^J \\ & - \tau_{k,t} \left[R_{k,t}^J - \delta_{K_t}^J P_t q_t^J \right] K_{t-1}^J(i) \ . \end{aligned}$$

Optimization is subject to the equality of output with demand

$$F(K_{t-1}^J(i), U_t^J(i), X_t^J(i)) = Z_t^J(i) \ , \ \text{where}$$

$$F(K_{t-1}^J(i), U_t^J(i), X_t^J(i)) =$$

$$\mathfrak{T} \left(\left(1 - \alpha_{J_t}^X\right)^{\frac{1}{\xi_{XJ}}} \left(M_t^J(i)\right)^{\frac{\xi_{XJ}-1}{\xi_{XJ}}} + \left(\alpha_{J_t}^X\right)^{\frac{1}{\xi_{XJ}}} \left(X_t^J(i) \left(1 - G_{X,t}^J(i)\right)\right)^{\frac{\xi_{XJ}-1}{\xi_{XJ}}} \right)^{\frac{\xi_{XJ}}{\xi_{XJ}-1}} \ ,$$

$$M_t^J(i) = \left(\left(1 - \alpha_J^U\right)^{\frac{1}{\xi_{ZJ}}} \left(K_{t-1}^J(i)\right)^{\frac{\xi_{ZJ}-1}{\xi_{ZJ}}} + \left(\alpha_J^U\right)^{\frac{1}{\xi_{ZJ}}} \left(T_t A_t^J U_t^J(i)\right)^{\frac{\xi_{ZJ}-1}{\xi_{ZJ}}} \right)^{\frac{\xi_{ZJ}}{\xi_{ZJ}-1}} \ .$$

$$Z_t^J(i) = \left(\frac{P_t^{\tilde{J}}(i)}{P_t^{\tilde{J}}} \right)^{-\sigma_J} Z_t^J \ .$$

We also have the following capital accumulation equation and adjustment costs:

$$K_t^J(i) = \left(1 - \delta_{K_t}^J\right) K_{t-1}^J(i) + S_t^{inv} I_t^J(i) \ ,$$

$$G_{P,t}^J(i) = \frac{\phi_{P^J}}{2} Z_t^J \left(\frac{\frac{P_t^{\tilde{J}}(i)}{P_{t-1}^{\tilde{J}}(i)}}{\frac{P_{t-1}^{\tilde{J}}}{P_{t-2}^{\tilde{J}}}} - 1 \right)^2 \ ,$$

$$G_{X,t}^J(i) = \frac{\phi_X^J}{2} \left(\frac{\left(X_t^J(i)/\left(gn\right)\right) - X_{t-1}^J}{X_{t-1}^J} \right)^2 \ ,$$

$$G_{U,t}^J(i) = \frac{\phi_U}{2} U_t^J \left(\frac{\left(U_t^J(i)/n\right) - U_{t-1}^J(i)}{U_{t-1}^J(i)} \right)^2 \ ,$$

$$G_{I,t}^J(i) = \frac{\phi_I}{2} I_t^J \left(\frac{\left(I_t^J(i)/(gn)\right) - I_{t-1}^J(i)}{I_{t-1}^J(i)} \right)^2 \ .$$

We write out the profit maximization problem of a representative manufacturing firm in Lagrangian form. Terms pertaining to period t and $t+1$ are sufficient. We introduce a multiplier Λ_t^J for the market clearing condition $F(K_{t-1}^J(i), U_t^J(i), X_t^J(i)) = \left(\frac{P_t^{\tilde{J}}(i)}{P_t^{\tilde{J}}}\right)^{-\sigma_J} Z_t^J$. The variable Λ_t^J equals the nominal marginal cost of producing one more unit of good i in sector J. We also introduce a multiplier q_t^J for the capital accumulation equation, which represents the shadow value of an additional unit of installed capital (Tobin's q) in terms of current investment goods. We have

$$\underset{P_s^{\tilde{J}}(i),U_s^J(i),I_s^J(i),K_{s+1}^J(i)}{Max} E_t \Sigma_{s=t}^{\infty} \tilde{R}_{t,s} D_{t+s}^J(i) = \tag{313}$$

$$\Bigg[\left(P_t^{\tilde{J}}(i)\right)^{1-\sigma_J}\left(P_t^{\tilde{J}}\right)^{\sigma_J} Z_t^J - V_t U_t^J(i) - P_t^X X_t^J(i) - P_t^I I_t^J(i)$$

$$-\tau_{k,t}\left(R_{k,t}^J - \delta_{K_t}^J P_t q_t^J\right) K_{t-1}^J(i) - P_t^{\tilde{J}} Z_t^J \frac{\phi_{P^J}}{2}\left(\frac{\frac{P_t^{\tilde{J}}(i)}{P_{t-1}^{\tilde{J}}(i)}}{\frac{P_{t-1}^{\tilde{J}}}{P_{t-2}^{\tilde{J}}}} - 1\right)^2 - P_t^{\tilde{J}} T_t \omega^J$$

$$-P_t^I \frac{\phi_I}{2} I_t^J \left(\frac{(I_t^J(i)/(gn)) - I_{t-1}^J(i)}{I_{t-1}^J(i)}\right)^2 - V_t \frac{\phi_U}{2} U_t^J \left(\frac{(U_t^J(i)/n) - U_{t-1}^J(i)}{U_{t-1}^J(i)}\right)^2\Bigg]$$

$$+\Lambda_t^J\left[F(K_{t-1}^J(i), U_t^J(i), X_t^J(i)) - P_t^{\tilde{J}}(i)^{-\sigma_J} P_t^{\tilde{J}\sigma_J} Z_t^J\right]$$

$$-q_t^J P_t\left[K_t^J(i) - (1-\delta_{K_t}^J) K_{t-1}^J(i) - S_t^{inv} I_t^J(i)\right]$$

$$+E_t\Bigg\{\frac{\theta\left(1+\xi_t^b\right)}{i_t}\Bigg[\left(P_{t+1}^{\tilde{J}}(i)\right)^{1-\sigma_J}\left(P_{t+1}^{\tilde{J}}\right)^{\sigma_J} Z_{t+1}^J - V_{t+1} U_{t+1}^J(i) - P_{t+1}^X X_{t+1}^J(i) - P_{t+1}^I I_{t+1}^J(i)$$

$$-\tau_{k,t+1}\left(R_{k,t+1}^J - \delta_{K_{t+1}}^J P_{t+1} q_{t+1}^J\right) K_t^J(i) - P_{t+1}^{\tilde{J}} Z_{t+1}^J \frac{\phi_{P^J}}{2}\left(\frac{\frac{P_{t+1}^{\tilde{J}}(i)}{P_t^{\tilde{J}}(i)}}{\frac{P_t^{\tilde{J}}}{P_{t-1}^{\tilde{J}}}} - 1\right)^2 - P_{t+1}^{\tilde{J}} T_{t+1} \omega^J$$

$$-P_{t+1}^I \frac{\phi_I}{2} I_{t+1}^J \left(\frac{(I_{t+1}^J(i)/(gn)) - I_t^J(i)}{I_t^J(i)}\right)^2 - V_{t+1} \frac{\phi_U}{2} U_{t+1}^J \left(\frac{(U_{t+1}^J(i)/n) - U_t^J(i)}{U_t^J(i)}\right)^2\Bigg]$$

$$+\frac{\Lambda_{t+1}^J \theta\left(1+\xi_t^b\right)}{i_t}\left[F(K_t^J(i), U_{t+1}^J(i), X_{t+1}^J(i)) - P_{t+1}^{\tilde{J}}(i)^{-\sigma_J} P_{t+1}^{\tilde{J}}{}^{\sigma_J} Z_{t+1}^J\right]$$

$$-\frac{q_{t+1}^J P_{t+1} \theta\left(1+\xi_t^b\right)}{i_t}\left[K_{t+1}^J(i) - (1-\delta_{K_{t+1}}^J) K_t^J(i) - S_{t+1}^{inv} I_{t+1}^J(i)\right]\Bigg\}$$

+terms pertaining to periods $t+2, t+3, ...$

We take the first-order condition with respect to $P_t^{\tilde{J}}(i)$ and then impose symmetry by setting $P_t^{\tilde{J}}(i) = P_t^{\tilde{J}}$ and $Z_t^J(i) = Z_t^J$ because all firms face an identical problem. We let $\lambda_t^J = \Lambda_t^J/P_t$ and rescale by technology. Then we obtain

$$\begin{aligned}\left[\frac{\sigma_J}{\sigma_J - 1}\frac{\lambda_t^J}{p_t^{\tilde{J}}} - 1\right] &= \frac{\phi_{P^J}}{\sigma_J - 1}\left(\frac{\pi_t^{\tilde{J}}}{\pi_{t-1}^{\tilde{J}}}\right)\left(\frac{\pi_t^{\tilde{J}}}{\pi_{t-1}^{\tilde{J}}} - 1\right) \\ &- E_t\frac{\theta g n}{\check{r}_t}\frac{\phi_{P^J}}{\sigma_J - 1}\left\{\frac{p_{t+1}^{\tilde{J}}}{p_t^{\tilde{J}}}\frac{\check{Z}_{t+1}^J}{\check{Z}_t^J}\left(\frac{\pi_{t+1}^{\tilde{J}}}{\pi_t^{\tilde{J}}}\right)\left(\frac{\pi_{t+1}^{\tilde{J}}}{\pi_t^{\tilde{J}}} - 1\right)\right\} .\end{aligned} \tag{314}$$

For $U_t^J(i)$, $X_t^J(i)$, $I_t^J(i)$, and $K_t^J(i)$ we have

$$\left(\frac{\lambda_t^J\check{F}_{U,t}^J}{\check{v}_t} - 1\right) = \phi_U\left(\frac{\check{U}_t}{\check{U}_{t-1}}\right)\left(\frac{\check{U}_t - \check{U}_{t-1}}{\check{U}_{t-1}}\right) - \frac{\theta g n}{\check{r}_t}\phi_U\frac{\check{v}_{t+1}}{\check{v}_t}\left(\frac{\check{U}_{t+1}}{\check{U}_t}\right)^2\left(\frac{\check{U}_{t+1} - \check{U}_t}{\check{U}_t}\right) , \tag{315}$$

$$p_t^X = \lambda_t^J\check{F}_{X,t}^J , \tag{316}$$

$$q_t^J S_t^{inv} = p_t^I + \phi_I p_t^I\left(\frac{\check{I}_t^J}{\check{I}_{t-1}^J}\right)\left(\frac{\check{I}_t^J - \check{I}_{t-1}^J}{\check{I}_{t-1}^J}\right) - \frac{\theta g n}{\check{r}_t}\phi_I p_{t+1}^I\left(\frac{\check{I}_{t+1}^J}{\check{I}_t^J}\right)^2\left(\frac{\check{I}_{t+1}^J - \check{I}_t^J}{\check{I}_t^J}\right) , \tag{317}$$

$$q_t^J = \frac{\theta}{\check{r}_t}E_t\left[q_{t+1}^J(1 - \delta_{K_{t+1}}^J) + r_{k,t+1}^J - \tau_{k,t+1}\left(r_{k,t+1}^J - \delta_{K_{t+1}}^J q_{t+1}^J\right)\right] , \tag{318}$$

where we have used

$$\check{F}_{U,t}^J = \mathcal{T}\left(\frac{\left(1 - \alpha_{J_t}^X\right)\check{Z}_t^J}{\mathcal{T}\check{M}_t^J}\right)^{\frac{1}{\xi_{XJ}}} A_t^J\left(\frac{\alpha_J^U\check{M}_t^J}{A_t^J\check{U}_t^J}\right)^{\frac{1}{\xi_{ZJ}}} , \tag{319}$$

$$\check{F}_{X,t}^J = \mathcal{T}\left(\frac{\alpha_{J_t}^X\check{Z}_t^J}{\mathcal{T}\check{X}_t^J\left(1 - G_{X,t}^J\right)}\right)^{\frac{1}{\xi_{XJ}}}\left(1 - G_{X,t}^J - \phi_X^J\frac{\check{X}_t^J}{\check{X}_{t-1}^J}\left(\frac{\check{X}_t^J - \check{X}_{t-1}^J}{\check{X}_{t-1}^J}\right)\right) , \tag{320}$$

$$r_{k,t}^J = \check{\lambda}_t^J\check{F}_{K,t}^J , \tag{321}$$

$$\check{F}_{K,t}^J = \mathcal{T}\left(\frac{\left(1 - \alpha_{J_t}^X\right)\check{Z}_t^J}{\mathcal{T}\check{M}_t^J}\right)^{\frac{1}{\xi_{XJ}}}\left(\frac{\left(1 - \alpha_J^U\right)\check{M}_t^J}{\check{K}_{t-1}^J}\right)^{\frac{1}{\xi_{ZJ}}} . \tag{322}$$

Appendix E. Manufacturers - With Financial Accelerator

The objective function facing each manufacturing firm in sectors $J \in \{N, T\}$ is

$$\underset{P_s^J(i),U_s^J(i),I_s^J(i),K_{s+1}^J(i)}{Max} E_t \Sigma_{s=t}^{\infty} \tilde{R}_{t,s} D_{t+s}^J(i) \ .$$

The price (and inflation) terms in the two sectors will be indexed with $\tilde{J} \in \{N, TH\}$. Then dividend terms are given by

$$\begin{aligned} D_t^J(i) &= P_t^{\tilde{J}}(i) Z_t^J(i) - V_t U_t^J(i) - P_t^X X_t^J(i) - R_{k,t}^J K_{t-1}^J(i) \\ &\quad - V_t G_{U,t}^J(i) - P_t^{\tilde{J}} G_{P,t}^J(i) - P_t^{\tilde{J}} T_t \omega^J \\ &\quad - \tau_{k,t} \left[R_{k,t}^J - \delta_{K_t}^J P_t q_t^J \right] K_{t-1}^J(i) \ . \end{aligned}$$

Optimization is subject to the equality of output with demand

$$F(K_{t-1}^J(i), U_t^J(i), X_t^J(i)) = Z_t^J(i) \ , \text{ where}$$

$$F(K_{t-1}^J(i), U_t^J(i), X_t^J(i)) =$$

$$\mathfrak{T} \left(\left(1 - \alpha_{J_t}^X\right)^{\frac{1}{\xi_{XJ}}} \left(M_t^J(i)\right)^{\frac{\xi_{XJ}-1}{\xi_{XJ}}} + \left(\alpha_{J_t}^X\right)^{\frac{1}{\xi_{XJ}}} \left(X_t^J(i) \left(1 - G_{X,t}^J(i)\right)\right)^{\frac{\xi_{XJ}-1}{\xi_{XJ}}} \right)^{\frac{\xi_{XJ}}{\xi_{XJ}-1}} \ ,$$

$$M_t^J(i) = \left(\left(1 - \alpha_J^U\right)^{\frac{1}{\xi_{ZJ}}} \left(K_{t-1}^J(i)\right)^{\frac{\xi_{ZJ}-1}{\xi_{ZJ}}} + \left(\alpha_J^U\right)^{\frac{1}{\xi_{ZJ}}} \left(T_t A_t^J U_t^J(i)\right)^{\frac{\xi_{ZJ}-1}{\xi_{ZJ}}} \right)^{\frac{\xi_{ZJ}}{\xi_{ZJ}-1}} \ .$$

$$Z_t^J(i) = \left(\frac{P_t^{\tilde{J}}(i)}{P_t^{\tilde{J}}} \right)^{-\sigma_J} Z_t^J \ .$$

We also have the following adjustment costs:

$$G_{P,t}^J(i) = \frac{\phi_{P^J}}{2} Z_t^J \left(\frac{\frac{P_t^{\tilde{J}}(i)}{P_{t-1}^{\tilde{J}}(i)}}{\frac{P_{t-1}^{\tilde{J}}}{P_{t-2}^{\tilde{J}}}} - 1 \right)^2 \ ,$$

$$G_{X,t}^J(i) = \frac{\phi_X^J}{2} \left(\frac{\left(X_t^J(i)/\left(gn\right)\right) - X_{t-1}^J}{X_{t-1}^J} \right)^2 \ ,$$

$$G_{U,t}^J(i) = \frac{\phi_U}{2} U_t^J \left(\frac{\left(U_t^J(i)/n\right) - U_{t-1}^J(i)}{U_{t-1}^J(i)} \right)^2 \ .$$

We write out the profit maximization problem of a representative manufacturing firm in Lagrangian form. Terms pertaining to period t and $t+1$ are sufficient. We introduce a multiplier Λ_t^J for the market clearing condition $F(K_{t-1}^J(i), U_t^J(i), X_t^J(i)) = \left(\frac{P_t^{\tilde{J}}(i)}{P_t^{\tilde{J}}}\right)^{-\sigma_J} Z_t^J$. The variable Λ_t^J equals the nominal marginal cost of producing one more unit of good i in sector J. We also introduce a multiplier q_t^J for the capital accumulation equation, which represents the shadow value of an additional unit of installed capital (Tobin's q) in terms of current investment goods. We have

$$\underset{P_s^{\tilde{J}}(i), U_s^J(i), I_s^J(i), K_{s+1}^J(i)}{Max} E_t \Sigma_{s=t}^{\infty} \tilde{R}_{t,s} D_{t+s}^J(i) = \tag{323}$$

$$\Bigg[\left(P_t^{\tilde{J}}(i)\right)^{1-\sigma_J}\left(P_t^{\tilde{J}}\right)^{\sigma_J} Z_t^J - V_t U_t^J(i) - P_t^X X_t^J(i) - R_{k,t}^J K_{t-1}^J(i)$$

$$-\tau_{k,t}\left(R_{k,t}^J - \delta_{K_t}^J P_t q_t^J\right) K_{t-1}^J(i) - P_t^{\tilde{J}} T_t \omega^J$$

$$-P_t^{\tilde{J}} Z_t^J \frac{\phi_{P^J}}{2}\left(\frac{\frac{P_t^{\tilde{J}}(i)}{P_{t-1}^{\tilde{J}}(i)}}{\frac{P_{t-1}^{\tilde{J}}}{P_{t-2}^{\tilde{J}}}} - 1\right)^2 - V_t \frac{\phi_U}{2} U_t^J \left(\frac{(U_t^J(i)/n) - U_{t-1}^J(i)}{U_{t-1}^J(i)}\right)^2\Bigg]$$

$$+\Lambda_t^J\left[F(K_{t-1}^J(i), U_t^J(i), X_t^J(i)) - P_t^{\tilde{J}}(i)^{-\sigma_J} P_t^{\tilde{J}\sigma_J} Z_t^J\right]$$

$$+E_t\Bigg\{\frac{\theta\left(1+\xi_t^b\right)}{i_t}\Bigg[\left(P_{t+1}^{\tilde{J}}(i)\right)^{1-\sigma_J}\left(P_{t+1}^{\tilde{J}}\right)^{\sigma_J} Z_{t+1}^J - V_{t+1} U_{t+1}^J(i) - P_{t+1}^X X_{t+1}^J(i) - R_{k,t+1}^J K_t^J(i)$$

$$-\tau_{k,t+1}\left(R_{k,t+1}^J - \delta_{K_{t+1}}^J P_{t+1} q_{t+1}^J\right) K_t^J(i) - P_{t+1}^{\tilde{J}} T_{t+1} \omega^J$$

$$-P_{t+1}^{\tilde{J}} Z_{t+1}^J \frac{\phi_{P^J}}{2}\left(\frac{\frac{P_{t+1}^{\tilde{J}}(i)}{P_t^{\tilde{J}}(i)}}{\frac{P_t^{\tilde{J}}}{P_{t-1}^{\tilde{J}}}} - 1\right)^2 - V_{t+1} \frac{\phi_U}{2} U_{t+1}^J \left(\frac{(U_{t+1}^J(i)/n) - U_t^J(i)}{U_t^J(i)}\right)^2\Bigg]$$

$$+\frac{\Lambda_{t+1}^J \theta\left(1+\xi_t^b\right)}{i_t}\left[F(K_t^J(i), U_{t+1}^J(i), X_{t+1}^J(i)) - P_{t+1}^{\tilde{J}}(i)^{-\sigma_J} {P_{t+1}^{\tilde{J}}}^{\sigma_J} Z_{t+1}^J\right]\Bigg\}$$

$$+\text{terms pertaining to periods } t+2, t+3, ...$$

We take the first-order condition with respect to $P_t^{\tilde{J}}(i)$ and then impose symmetry by setting $P_t^{\tilde{J}}(i) = P_t^{\tilde{J}}$ and $Z_t^J(i) = Z_t^J$ because all firms face an identical problem. We let $\lambda_t^J = \Lambda_t^J / P_t$ and rescale by technology. Then we obtain

$$\left[\frac{\sigma_J}{\sigma_J - 1}\frac{\lambda_t^J}{p_t^{\tilde{J}}} - 1\right] = \frac{\phi_{P^J}}{\sigma_J - 1}\left(\frac{\pi_t^{\tilde{J}}}{\pi_{t-1}^{\tilde{J}}}\right)\left(\frac{\pi_t^{\tilde{J}}}{\pi_{t-1}^{\tilde{J}}} - 1\right) \tag{324}$$

$$-E_t \frac{\theta g n}{\check{r}_t}\frac{\phi_{P^J}}{\sigma_J - 1}\left\{\frac{p_{t+1}^{\tilde{J}}}{p_t^{\tilde{J}}}\frac{\check{Z}_{t+1}^J}{\check{Z}_t^J}\left(\frac{\pi_{t+1}^{\tilde{J}}}{\pi_t^{\tilde{J}}}\right)\left(\frac{\pi_{t+1}^{\tilde{J}}}{\pi_t^{\tilde{J}}} - 1\right)\right\}.$$

For $U_t^J(i)$, $X_t^J(i)$, and $K_t^J(i)$ we have

$$\left(\frac{\lambda_t^J \check{F}_{U,t}^J}{\check{v}_t} - 1\right) = \phi_U \left(\frac{\check{U}_t}{\check{U}_{t-1}}\right)\left(\frac{\check{U}_t - \check{U}_{t-1}}{\check{U}_{t-1}}\right) - \frac{\theta g n}{\check{r}_t} \phi_U \frac{\check{v}_{t+1}}{\check{v}_t}\left(\frac{\check{U}_{t+1}}{\check{U}_t}\right)^2 \left(\frac{\check{U}_{t+1} - \check{U}_t}{\check{U}_t}\right) \ , \tag{325}$$

$$p_t^X = \lambda_t^J \check{F}_{X,t}^J \ , \tag{326}$$

$$r_{k,t}^J = \check{\lambda}_t^J \check{F}_{K,t}^J \ , \tag{327}$$

where we have used

$$\check{F}_{U,t}^J = \mathcal{T}\left(\frac{\left(1-\alpha_{J_t}^X\right)\check{Z}_t^J}{\mathcal{T}\check{M}_t^J}\right)^{\frac{1}{\xi_{XJ}}} A_t^J \left(\frac{\alpha_J^U \check{M}_t^J}{A_t^J \check{U}_t^J}\right)^{\frac{1}{\xi_{ZJ}}} \ , \tag{328}$$

$$\check{F}_{X,t}^J = \mathcal{T}\left(\frac{\alpha_{J_t}^X \check{Z}_t^J}{\mathcal{T}\check{X}_t^J\left(1-G_{X,t}^J\right)}\right)^{\frac{1}{\xi_{XJ}}}\left(1 - G_{X,t}^J - \phi_X^J \frac{\check{X}_t^J}{\check{X}_{t-1}^J}\left(\frac{\check{X}_t^J - \check{X}_{t-1}^J}{\check{X}_{t-1}^J}\right)\right) \ , \tag{329}$$

$$\check{F}_{K,t}^J = \mathcal{T}\left(\frac{\left(1-\alpha_{J_t}^X\right)\check{Z}_t^J}{\mathcal{T}\check{M}_t^J}\right)^{\frac{1}{\xi_{XJ}}}\left(\frac{\left(1-\alpha_J^U\right)\check{M}_t^J}{\check{K}_{t-1}^J}\right)^{\frac{1}{\xi_{ZJ}}} \ . \tag{330}$$

Appendix F. Entrepreneur's Problem - Lognormal Distribution

Basic Properties of Γ and G

We first repeat the expressions for Γ and G here for ease of reference:

$$\Gamma(\bar{\omega}^J_{t+1}) \equiv \int_0^{\bar{\omega}^J_{t+1}} \omega^J_{t+1} f(\omega^J_{t+1}) d\omega^J_{t+1} + \bar{\omega}^J_{t+1} \int_{\bar{\omega}^J_{t+1}}^{\infty} , \tag{331}$$

$$G(\bar{\omega}^J_{t+1}) = \int_0^{\bar{\omega}^J_{t+1}} \omega^J_{t+1} f(\omega^J_{t+1}) d\omega^J_{t+1} . \tag{332}$$

Then we have

$$\Gamma^{'}_{J,t+1} = 1 - F(\bar{\omega}^J_{t+1}) , \tag{333}$$

$$G^{'}_{J,t+1} = \bar{\omega}^J_{t+1} f(\bar{\omega}^J_{t+1}) . \tag{334}$$

Basic Properties of the Lognormal Distribution

The assumption is that ω^J_t is lognormally distributed with $E(\omega^J_t) = 1$ and $Var(\omega^J_t) = \left(\sigma^J_t\right)^2$. This implies the following:

$$\ln(\omega^J_t) \sim N(-\frac{1}{2}\left(\sigma^J_t\right)^2, \left(\sigma^J_t\right)^2) , \tag{335}$$

$$f(\omega^J_t) = \frac{1}{\sqrt{2\pi}\omega^J_t \sigma^J_t} \exp\left\{-\frac{1}{2}\left(\frac{\ln(\omega^J_t) + \frac{1}{2}\left(\sigma^J_t\right)^2}{\sigma^J_t}\right)^2\right\} . \tag{336}$$

Derivations

We will change integrands at various points in order to obtain solutions that can be expressed in terms of the cumulative distribution function Φ of the standard normal distribution. We begin by defining terms:

$$\bar{z}^J_t = \frac{\ln(\bar{\omega}^J_t) + \frac{1}{2}\left(\sigma^J_t\right)^2}{\sigma^J_t} \quad , \quad y^J_t = \frac{\ln(\omega^J_t) + \frac{1}{2}\left(\sigma^J_t\right)^2}{\sigma^J_t} , \tag{337}$$

$$\tilde{z}^J_t = \frac{\ln(\bar{\omega}^J_t) - \frac{1}{2}\left(\sigma^J_t\right)^2}{\sigma^J_t} \quad , \quad \tilde{y}^J_t = \frac{\ln(\omega^J_t) - \frac{1}{2}\left(\sigma^J_t\right)^2}{\sigma^J_t} . \tag{338}$$

Manipulating the second expression in each case gives the following expressions:

$$d\omega^J_t = \sigma^J_t \exp\left\{y^J_t \sigma^J_t - \frac{1}{2}\left(\sigma^J_t\right)^2\right\} dy^J_t , \tag{339}$$

$$d\omega^J_t = \sigma^J_t \exp\left\{y^J_t \sigma^J_t + \frac{1}{2}\left(\sigma^J_t\right)^2\right\} d\tilde{y}^J_t , \tag{340}$$

Using (336)-(340) we can now evaluate the expressions determining Γ and G in terms of the c.d.f. $\Phi(.)$. We start with

$$\int_{\bar{\omega}_{t+1}^J}^{\infty} f(\omega_{t+1}^J)d\omega_{t+1}^J = \int_{\bar{\omega}_{t+1}^J}^{\infty} \frac{1}{\sqrt{2\pi}\omega_{t+1}^J\sigma_{t+1}^J} \exp\left\{-\frac{1}{2}\left(\frac{\ln(\omega_{t+1}^J)+\frac{1}{2}\left(\sigma_{t+1}^J\right)^2}{\sigma_{t+1}^J}\right)^2\right\} d\omega_{t+1}^J$$

$$= \int_{\bar{z}_{t+1}^J}^{\infty} \frac{\sigma_{t+1}^J}{\sqrt{2\pi}\omega_{t+1}^J\sigma_{t+1}^J} \exp\left\{-\frac{1}{2}\left(y_{t+1}^J\right)^2\right\} \exp\left\{y_{t+1}^J\sigma_{t+1}^J - \frac{1}{2}\left(\sigma_{t+1}^J\right)^2\right\} dy_{t+1}^J$$

$$= \int_{\bar{z}_{t+1}^J}^{\infty} \frac{1}{\sqrt{2\pi}} \frac{1}{\omega_{t+1}^J} \exp\left\{-\frac{1}{2}\left(\left(y_{t+1}^J\right)^2 + \left(\sigma_{t+1}^J\right)^2 - 2y_{t+1}^J\sigma_{t+1}^J\right)\right\} dy_{t+1}^J$$

$$= \int_{\bar{z}_{t+1}^J}^{\infty} \frac{1}{\sqrt{2\pi}} \frac{1}{\omega_{t+1}^J} \exp\left\{-\frac{1}{2}\left(y_{t+1}^J - \sigma_{t+1}^J\right)^2\right\} dy_{t+1}^J$$

$$= \int_{\bar{z}_{t+1}^J}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left\{-\ln(\omega_{t+1}^J)\right\} \exp\left\{-\frac{\left(\ln(\omega_t^J) - \frac{1}{2}\left(\sigma_{t+1}^J\right)^2\right)^2}{2\left(\sigma_{t+1}^J\right)^2}\right\} dy_{t+1}^J$$

$$= \int_{\bar{z}_{t+1}^J}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left\{\frac{-2\left(\sigma_{t+1}^J\right)^2\ln(\omega_{t+1}^J) - \left(\ln(\omega_t^J)\right)^2 - \left(\frac{1}{2}\left(\sigma_{t+1}^J\right)^2\right)^2 + \ln(\omega_t^J)\left(\sigma_{t+1}^J\right)^2}{2\left(\sigma_{t+1}^J\right)^2}\right\} dy_{t+1}^J$$

$$= \int_{\bar{z}_{t+1}^J}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{\left(\ln(\omega_t^J)\right)^2 + \left(\frac{1}{2}\left(\sigma_{t+1}^J\right)^2\right)^2 + 2\ln(\omega_{t+1}^J)\frac{1}{2}\left(\sigma_{t+1}^J\right)^2}{2\left(\sigma_{t+1}^J\right)^2}\right\} dy_{t+1}^J$$

$$= \int_{\bar{z}_{t+1}^J}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{\ln(\omega_t^J)+\frac{1}{2}\left(\sigma_{t+1}^J\right)^2}{\sigma_{t+1}^J}\right)\right\} dy_{t+1}^J$$

$$= \int_{\bar{z}_{t+1}^J}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}y_{t+1}^J\right\} dy_{t+1}^J = 1 - \Phi\left(\bar{z}_{t+1}^J\right) \ .$$

Next we have

$$\int_{\bar{\omega}_{t+1}^J}^{\infty} \omega_{t+1}^J f(\omega_{t+1}^J)d\omega_{t+1}^J = \int_{\bar{\omega}_{t+1}^J}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_{t+1}^J} \exp\left\{-\frac{1}{2}\left(\frac{\ln(\omega_{t+1}^J)+\frac{1}{2}\left(\sigma_{t+1}^J\right)^2}{\sigma_{t+1}^J}\right)^2\right\} d\omega_{t+1}^J$$

$$= \int_{\tilde{z}_{t+1}^J}^{\infty} \frac{\sigma_{t+1}^J}{\sqrt{2\pi}\sigma_{t+1}^J} \exp\left\{-\frac{1}{2}\left(\tilde{y}_{t+1}^J + \sigma_{t+1}^J\right)^2\right\} \exp\left\{\tilde{y}_{t+1}^J\sigma_{t+1}^J + \frac{1}{2}\left(\sigma_{t+1}^J\right)^2\right\} d\tilde{y}_{t+1}^J$$

$$= \int_{\tilde{z}_{t+1}^J}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\tilde{y}_{t+1}^J\right)^2 - \frac{1}{2}\left(\sigma_{t+1}^J\right)^2 - \tilde{y}_{t+1}^J\sigma_{t+1}^J + \tilde{y}_{t+1}^J\sigma_{t+1}^J + \frac{1}{2}\left(\sigma_{t+1}^J\right)^2\right\} d\tilde{y}_{t+1}^J$$

$$= \int_{\tilde{z}_{t+1}^J}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\tilde{y}_{t+1}^J\right)^2\right\} d\tilde{y}_{t+1}^J = 1 - \Phi\left(\tilde{z}_{t+1}^J\right) = 1 - \Phi\left(\bar{z}_{t+1}^J - \sigma_{t+1}^J\right)$$

To summarize:

$$\int_{\bar{\omega}_{t+1}^J}^{\infty} f(\omega_{t+1}^J) d\omega_{t+1}^J = 1 - \Phi\left(\bar{z}_{t+1}^J\right) \ , \tag{341}$$

$$\int_{\bar{\omega}_{t+1}^J}^{\infty} \omega_{t+1}^J f(\omega_{t+1}^J) d\omega_{t+1}^J = 1 - \Phi\left(\bar{z}_{t+1}^J - \sigma_{t+1}^J\right) \ . \tag{342}$$

Final Equation System

The entrepreneur's optimal loan contract condition (98) determines the equilibrium return to capital $r\check{e}t_{k,t}^J$, the lender's zero profit condition (99) determines the lender's gross profit share Γ_{t+1}^J, and the net worth accumulation condition (107) determines the entrepreneur's net worth $\check{n}_t^J$. The conditions derived in this appendix close the system. To summarize, we have:

$$\bar{z}_t^J = \frac{\ln(\bar{\omega}_t^J) + \frac{1}{2}\left(\sigma_t^J\right)^2}{\sigma_t^J} \ , \tag{343}$$

$$f\left(\bar{\omega}_t^J\right) = \frac{1}{\sqrt{2\pi}\bar{\omega}_t^J \sigma_t^J} \exp\left\{-\frac{1}{2}\left(\bar{z}_t^J\right)^2\right\} \ , \tag{344}$$

$$\Gamma_t^J = \Phi\left(\bar{z}_t^J - \sigma_t^J\right) + \bar{\omega}_t^J\left(1 - \Phi\left(\bar{z}_t^J\right)\right) \ , \tag{345}$$

$$G_t^J = \Phi\left(\bar{z}_t^J - \sigma_t^J\right) \ , \tag{346}$$

$$\Gamma'_{J,t} = 1 - \Phi\left(\bar{z}_t^J\right) \ , \tag{347}$$

$$G'_{J,t} = \bar{\omega}_t^J f\left(\bar{\omega}_t^J\right) \ . \tag{348}$$